Mathematics, mathematics education and war

Bernhelm Booss-Bavnbek, Roskilde University, Denmark

On the third evening of the conference there was a post-dinner meeting: An "agora" discussing "Mathematics and War". The term "agora" stems from ancient Greece and means "market place" or also "the meeting of the people". It was the intention of the organisers that the meeting should be held in a non-rigidly organised manner, letting the participants freely discuss the topic. The agora was initiated by Jessica Carter (Odense University, Denmark) and directed by Dimitris Chassapis (Aristotle University of Thessaloniki, Greece). I was asked to open the meeting by giving a 20 minutes talk. Here is what I said.

Thank you for the opportunity to share with you some concern regarding the ongoing and accelerating militarisation of our subject, mathematics, and the persisting silence of math educators about it. I shall address the interrelations between mathematics and military from three points of view:

Military traces in the history of mathematics and in present-day mathematics.

Changes in the character of war under the influence of mathematical thinking, mathematical methods and results, and mathematics supported technology.

Best guess for explaining the silence of the math educators.

My talk is based on my own math teaching at Roskilde University (Denmark); on a book I have written (with the math historian Jens Høyrup)[1] and on the materials of a conference on the subject to be held in Karlskrona (Sweden).[2]

It is late. You must be tired after so many lectures in a densely packed programme. I will show you only one transparency. It is the frontispiece of Niccolò Tartaglia's *Nova scientia*, Venice, 1537. Today in our math history books, Tartaglia is mostly remembered for his solution of the algebraic equation of third degree. But on top we read his Latin "welcome to what is proved by fire and mathematical ingenuity". Under the philosophers' (a little desolate) garden with the door keepers Plato and Aristotle, preventing the entrance to all not knowledgeable in geometry, we see the mathematised world, the door kept open

---

[1]B. Booss, J. Høyrup, Von Mathematik und Krieg, Marburg, 1984; English translation in J. Høyrup, In Measure, Number, and Weight. Studies in Mathematics and Culture, State University of New York Press, 1994, pp. 225-278 + notes, bibliography.

[2]B. Booss-Bavnbek, J. Høyrup, S.A. Pedersen (eds.), Mathematics and War. Proceedings of an International Scientific Meeting (Karlskrona, Sweden, August 2002), Birkhäuser Verlag, Basel and Boston, (to appear in spring 2003), ca. 300 pp.

by Euclid, with all the Muses and Liberal Arts. We see Musica, Arithmetica, Geometrica, and Astronomia. And we see two canons, one shooting flat, the other shooting steep with wonderful trajectories of the ball-formed projectiles.

QuickTime™ and a
Photo - JPEG decompressor
are needed to see this picture.

This was all propaganda. OK, Tartaglia wrote that he had found a mathematical theory to calculate the range of a canon from the elevation of the gun pipe; that he hat kept silent about his "rules for the art of the bombardier" for years because he did not want "to study and improve such a damnable exercise, destroyer of the human species, and especially of Christians in their continual wars"; and that he had to speak out now when the Turks were threatening Venice. As a matter of fact, this was perhaps the first time that the modern concept of a function or a curve was expressed.

However, Tartaglia's theory was still too rudimentary to become a true help for the bombardier working with very contingent ordnance technology. But it did its propaganda work: it promised additional reputation to mathematicians, teaching at state financed schools and universities, and perhaps it gave also more confidence of victory to troops on vessels and battle ground.

Looking through history we realise that until quite recent times (say Second World War) mathematics has not been markedly shaped by military demands in spite of the fact that so many mathematicians and math teachers throughout history have been financed by or for the military. We also see that until quite recent times (say late 1800s) warfare was not sufficiently industrialised to gain from mathematical supported technology on a large scale.

But from Tartaglia onwards we find that mathematicians and mathematics teachers have played a role either by claiming a decisive role of math technology in warfare or by preaching superiority and invincibility over less developed opponents by mathematical means, or by neglecting their professional duty to stand up against beliefs of math related superiority or invincibility in the classroom.

Let us have a closer look at the facts.

Military traces in the history of mathematics and in present day mathematics

In 1939, the British crystallographer and science historian John Desmond Bernal wrote: "Science and warfare have always been most closely linked; in fact, except for a certain portion of the nineteenth century, it may be fairly claimed that the majority of significant technical and scientific advances owe their origin directly to military or naval requirements."

This is to some extent also true for mathematics, though the mutual relation between military demands and emerging mathematical concepts is intricate. A few examples:

Babylonian clay tablets from about 1800 B.C. deal with *siege computation*: the number of bricks needed for siege ramps, the volume of earth to be dug and how much workforce was required. But the same calculations were used when building a temple or digging an irrigation canal. Moreover, many "real-world" mathematical problems on the thousands of preserved tablets show (for instance in the choice of unknowns for the quadratic equations) that they were meant to puzzle and train the student rather than to contribute to the solution of problems in real situations.

In spite of its declared praxis-remote motivation, *Euclidean geometry* has for two millennia trained geometric judgement and modelling capacity of immediate military relevance. The graecist John Onians traces the origin of Greek mathematics to “the importance of absolute order in the military sphere which gave mathematics a dominant role in all Greek culture”. An overlooked aspect of the present decline of Euclidean geometry in school curricula are the diminishing demands for distributed independent geometric calculations in the electronic battlefield.

Archimedes (ca. 287-212 B.C.) demonstrated during the siege of Syracuse his command of the full geometry of *three-dimensional translations and rotations* and of precise volume and weight estimates for plates, bodies and tools of a most delicate shape. His engineering calculations were admired, the artefacts copied and the proportions disseminated through all the Hellenistic, late Roman, and Arab times, and through the Middle Ages.

The military writer Aeneas Tacticus (around 360 B.C.) published a whole chapter with ideas about cryptology (for instance sending a book with one letter thinly marked in each line - an early instant of steganography). Subsequently Polybios, Caesar, and Augustus introduced true bilateral substitutions, i.e. *permutations* (of vowels and consonants) for military communication. The Islamic Middle Ages took over this complex of applied mathematics and (occasionally military inspired) techniques, expanded and moulded it into *arithmetic and algorithmic form* – like moving squadrons and armies - and handed it over to the early Modern period.

The Renaissance had a high appreciation of the possibilities of mathematics in every practice. Specific military needs (cartography, artillery and ballistics) partly preceded, partly met specific civilian needs (the theory of the central perspective, bookkeeping, merchants' calculation and algebra). Keeping it a strict military secret, the fifteenth-century Portuguese court took up a systematic development of *navigational mathematics* which led them to distinguish between great circle arcs (= geodetic curves) and loxodromes (curves of constant angles like the parallels). As mentioned before, when Niccolò Tartaglia (1500?-1557) tried to 'give rules for the art of the bombardier' (Nova scientia, 1537), he abandoned Aristotle's concept of a piecewise linear trajectory and created the modern concept of a *function* with a smoothly curved graph (though not yet the correct parabola orbit, later derived by Galileo). The Flemish mathematician Simon Stevin (1548-1620), later quartermaster general of the army under Maurice of Nassau, engineered a system of sluices to flood certain areas in defence of besieged cities and thus founded the modern *statics and hydrostatics* (De Beghinselen der Weeghconst, 1586).

Towards modern times

All these germs were systematically investigated in the following centuries, sometimes directly on military order like Harriot's practical and theoretical work on cartography (around 1585); Hooke's study of elasticity, ordered by the English Royal Society on behalf of the Navy which wanted to cut down the consumption of wood in shipbuilding; the construction of accurate chronometers and, alternatively, the creation of a precise theory of the lunar movement for precise empirical determination of geographical latitude; the fortification mathematics of Sébastien le Prestre, Marquis de Vauban, optimising the complex polygonal fortification structures with Monge's descriptive geometry as spin-off; the introduction of infinitesimal mathematics into the military curriculum to consider the effect of air resistance on the velocity of a projectile (then an impossible task). Forts and Howitzers were on the frontispieces of almost any mathematics book of the 16th to 18th century, even when they dealt solely with pure mathematics.

The practice of war, however, had no important influence on the development of mathematics: practical military or civilian preoccupations of that time proved fruitful for the development of mathematics and mathematics based physics and engineering only when they were related to the network of current scientific knowledge and theory. This is illustrated by the life, work, and lasting influence of the mathematical heroes of that time, Huygens, Fermat, Galileo, Newton, Leibniz, the Bernoullis, Euler, d'Alembert, Laplace, Gauss, Cauchy, Riemann, Poincaré, Hilbert, none of them very eager in military matters.

Gradually, mathematics as an academic discipline *itself* emerged in the sense of systematic research attached to universities, institutes of technology, and other institutions of higher learning. Only now could the reached maturation of mathematical knowledge lead to direct industrial applicability and by this to enhancing the military strength of a country. Prerequisites for this maturation were, in addition to an immense quantitative growth, a complete reorganisation of knowledge and a far-ranging division of intellectual labour. Physics and engineering sciences were organised independently. Pure and applied mathematics were separated from each other conceptually and institutionally.

Military traces in 20th Century mathematics

On the eve of and during World War I there were various attempts at organising technical development on a large-scale scientific basis. The general picture, however, was different: the French Supreme Command did not find a better use of its young scientific and technical talents than sending them into the trenches, while the German Supreme Command widely granted leave of absence for continuing research, also basic research. The intention was to save scientific manpower and thus preserve the basis for post-war technical development.

The Second World War brought a new alliance between mathematicians and the military, leaving a triad of complex artefacts behind: the atomic bomb with its stationary model in service of energy production; the jet propulsion for rockets, intercontinental bombers and charter travel; and the computer. Less visible was the creation of the new mathematical sciences of operations research (P.M.S. Blackett, G. B. Dantzig), coding (A. Turing), statistical quality control (A. Wald), signal processing and time series analysis (N. Wiener, C.E. Shannon, A. Kolmogorov), simulation and scientific computing (S. Ulam), numerical weather prediction (J. v. Neumann).

The complexity of artefacts, of production processes and of globalised social relations had their break-through in the aforementioned triad of deadly products, which the new alliance between mathematicians and warlords left behind after World War II. A new faith in the omnipotence of computer supported modelling emerged. Description, prediction and prescription (design) became based on a universal belief in the conversion of complex situations and problems into simplified and seemingly controllable perceptions.

According to Carl von Clausewitz, the brilliant war historian and war theoretician, such conversion is intimately related to the essence of war. But it is quite opposite to traditional mathematics taking pride in emphasising differences between the perceived and real complexity of a problem.

At the heart of it all is the electronic computer, which supports the operation of processes beyond the grasp of human minds using vast collections of software whose magnitude and complexity put it beyond the reach of reliability. Consider for instance the steering of nuclear power plants or of organisations of a size which need world markets for their sustenance. This blurs the borderlines between what we do and do not understand, what we can make and what we can control.

The successful co-operation between mathematicians and the military during World War II and the subsequent arms race let mathematics expand grossly as a science devoted to complexity and simplification. In that process, our profession was and is so much success oriented that essential quality marks of traditional mathematics, the discussion of the range, validity and consequences of our calculations and derivations, are often sacrificed.

The effects of mathematics on war

Sporadic evidence

Through history there are a few instances where we can point to a single mathematical calculation, concept or a mathematics-based construction of immediate military relevance.

Polybios judged it necessary to imprint Roman commanders that the hypotenuse is substantially longer than even the longest other side in a rectangular triangle and hence *ladders* for storming a wall should be substantially longer than the shear height of the wall.

Archimedes' mechanical calculations could not prevent the fall of Syracuse but supported the excellence of *siege machinery* in Roman warfare.

The application of *ordnance* (canons, howitzers) became efficient only with quantitative relations between the slope of the pipe and the range (though with empirical tables more reliable than theoretical deductions up till the 1940s).

We are used to attaching Galileo's work in geometrical optics for better *telescopes* to the confirmation of the new planetary view by the discovery of the Jupiter moons, but that was not what Galileo demonstrated to the Venice Seniors from the top of the tower of St. Marc. What he demonstrated was the military relevant capacity of seeing ships and their details long before encountering them.

Elaborated astronomical calculations of *astrology* decided the opening of quite a number of battles up till Wallenstein in the Thirty Years War in Germany.

A single calculation almost "on the back of an envelope", Peierl's and Frisch's 1940 estimation for the *critical mass* of U_235 to be less than 10 kg to maintain the fission process, convinced first the British and then the US government of the feasibility of the *atomic bomb* - contrary to Heisenberg, who did not do this calculation and, according to various testimony even after August 6, 1945 remained convinced that rather a whole metric ton of U_235 was needed for one single atomic bomb.

Another calculation (of 1943), again by Peierls and Frisch, who had then moved to Los Alamos, showed the feasibility of the *Plutonium bomb,* namely how conventional explosives had to be proportioned and placed around a sub-critical Pu ball to generate those shock waves which bring the Pu together into a critical mass - instead of blowing it up (details are still not disclosed).

According to rumours even more advanced calculations (mostly numerical simulations) were performed by Ulam and v. Neumann for nuclear branching processes and shock waves of explosives - now also involving fission bombs - as a triggering device for thermonuclear fusion, the *hydrogen bomb*.

A legendary case of mathematical ingenuity was Turing's contribution to the design of the "Colossi" computers and the *breaking of the German code* Enigma, by the Germans believed un-breakable until the end of the war.

The list of war-relevant mathematical ingenuity on the eve of and during World War II could be largely expanded. However, the question remains: were any mathematical efforts decisive for war?

Historians of World War II tend to conclude:

The enormous achievements in air plane construction, air raid operation and strategic bombing were, by large, according to the post-war Allied Bombing Survey, not very effective for diminishing the adversary's capacity to continue the war or shortening the war.

The two atomic bombs on Hiroshima and Nagasaki have clearly triggered the final Japanese capitulation. However, negotiations about the surrender were already on the way and the immediate military relevance was negligible compared to the Japanese breakdown on all fronts.

The decisive importance of the code breaking by the "Colossi" is attested for the naval war, mainly positioning of and defence against German submarines. It also provided some information for the land battle, but apparently not of real operational relevance. Signal intelligence rapidly expanded to an "industry" of some 30.000 people engaged on all sides, though without any further certified true war relevance. In recent wars and violence (for instance in Iraq, Near East, Kosovo, Afghanistan) not many coups were attributed to passive electronic surveillance and not one single coup to its active counterpart, cyberwar, though enormous means have been invested and though it has attracted enormous public interest.

Perhaps Lev Semyonovich Pontryagin's (1908-1988) mathematical work has had the greatest influence on peace and war in the 20th century. His mathematical career began with quite outstanding achievements in pure mathematics: The Pontryagin numbers for instance laid ground to modern global analysis and differential geometry and came to inspire research in algebraic geometry and number theory up till our days. In the late 40s he changed his field to the control theory of dynamical systems and became, especially with his "Maximum Principle", the mathematical father of trajectory control of *satellites* and *intercontinental ballistic missiles*. Providing the Soviet Union with means of intercontinental nuclear retaliation he contributed to putting an end to a protracted period of British and American nuclear war preparation. Until 1949 (mainly) London wanted to exploit the nuclear monopoly and from 1950 until 1955 (mainly) Washington wanted to use the advantage of B-36 and B-47 encirclement for nuclear black mailing and possible physical extinction of the Soviet Union.

A variety of mathematical algorithms are implemented in modern *terminal guidance systems* for missiles and bombs. The most important techniques are terrain contour matching by Fast Fourier Transform, Laser guidance by Fourier optics, satellite based Global Positioning System, precision timing for free fall bomb dropping. They have reduced the Circular Error Probable (CEP) from 1100 m in World War II to 13 m by the turn of the century. Here CEP denotes the radius of a disc around the goal point such that (on average) 50% of the bombs hit inside the disc.

C4ISR is the acronym for modern military *signal processing*. It stands for Control, Command, Communications, Computers, Intelligence, Surveillance, and Reconnaissance, that is to gather, process, hide, and disturb a huge variety of information about own and adversary forces.

From the point of mathematics also *stealth* is a kind of signal processing = reducing radar reflection by shaping the surface of a flying body in a particular way, namely moving eigenvalues from the resonance side to the non-resonance side.

The military use of *virtual reality* visualisation techniques can be characterised as "auto-stealth" since the point is to hide confusing or unwanted information for personnel in combat, commanders and the public.

Some military relevance may or may not be attributed to various types of *war games*, *combat simulation* and *dynamical models of arms race*. Critique will point to the self-deception aspects of this kind of modelling. However, the founder of numerical weather prediction and of numerical analysis at large, Lewis F. Richardson (1881-1953), himself an ardent pacifist, holds it that rigorous mathematical thinking about conflict never can do harm.

Systematic aspects

What changes? Roman warfare and Modern warfare since Machiavelli, Maurice of Nassau, Gustavus II Adolphus, and, foremost, the Napoleonic Wars and the World Wars, have been characterised by the rationality of goals and means; quantisation of troops, inventory, distances, order of battle; and discipline and shared goals of fighting troops, commanders and the hinterland.

Today, most military historians agree that the rise of modern war in the period 1500-1945 was accompanied by a rise in mathematical technology and other technical innovation, but not driven by it. Rather, the new ways of warfare came from the mathematical idea: "to liken war to business competition" (Clausewitz). Discarding differences and emotions between adversaries, the basic assumption of modern warfare is that both sides are guided by the same kind of logic, rationality, and reason.

Acknowledging an adversary's logic and reason has also been a decisive step in seeing him as an equal and finally in developing international law (for instance Grotius' claim to think about war "more geometrico") and international humanitarian law (for instance the Red-Cross concept of protection of equally valuable lives and civil objects).

In all that period, roughly speaking, warfare became gradually more calculable, whereas war itself, this grandiose piece of psychology, chaos and destruction, remained incalculable with a principally unpredictable outcome.

Accompanied - or driven - by the aforementioned new mathematical technology (this must be decided separately in each instant), the character of war, preparing for war and executing war have changed *after* World War II.

Irrational concepts like *nuclear genocide threat* or collective nuclear suicide are considered political and military options. There is no rational human argument about nuclear war or a nuclear first strike capacity.

However, on the positive side, the *outcome* of nuclear war became so predictable that it could support the principal of "Mutually Assured Destruction". In a period of international tension (in the 1960s and 70s) this paved the way for at least psychologically important talks and agreements between the two nuclear superpowers of that era. Practically, nuclear war has been averted hitherto.

Perceived predictability is also behind the newly emerging concept of *Hit'nd Run Wars*, based on a perceived full control of risks and accepted casualties and attributed to mathematical advances and revolutionary changes in mathematical technology (first of all ISR and precision ammunition). Examples are the recent *HighTech-(Own)LowCasualties* wars against Iraq, Yugoslavia and Afghanistan which resume the asymmetric warfare of the past colonial wars and wars against the American Indians.

The promise of a "limited" character of these wars is dangerous because it lowers the barriers of waging a war, which may not stay as limited and clean as planned.

However, the limited punishment by Hit'nd Run actions is no longer the privilege of mathematical technology supported former colonial masters. It may become a habit also among segments of the world population which do not identify with the present day masters and are willing to sacrifice their lives in LowTech-ManyCasualities *terrorist attacks* against centres of the dominant world part (case WTC).

Due to the real limitations of nature and natural laws it is very hard to invent and design new ways to kill, hurt, and disable humans or to destroy buildings and other human artefacts. The *chemistry* of energy storage has already been optimised during WWI. There *is* nothing "better" than TNT. The *physics* of energy release has since then been optimised during and shortly after WWII: there *is* nothing "better" than the atomic bomb and the hydrogen bomb. *Engineering* has provided better and better materials for the hardening of projectile tips and for strengthening tank armours. But this, too, is a principally limited process of metallurgic advance.

Now the special capacity of *mathematics* in modern armament and warfare is its role in enhancing the efficiency of delivery (traditionally the domain of engineering, for instance the construction of canons, tanks, and warhead enforcement). Here there are no natural limitations because we progress mainly through new ways of symbolic manipulation. In such a way mathematics promises an infinite curve of innovations in precision; timing and co-ordination; miniaturisation; new physical ways of delivery (case sound beam); and, worst of all, new dreams of invincibility.

Mathematical ethics and persisting silence of math educators.

To sum up, our findings are:

Math + rationalisation = lowering the risks of war;

Math + technology = ambiguous, as usual;

Math + ideology = poisonous superiority and invincibility illusion.

Instead of feeling the obligation to disseminate mathematical knowledge unconditionally we must develop the feeling of an obligation to hinder inconsiderate proliferation: every pupil, every student who falls to trust the media that waging war is no a clean business with pin-point precision, without bloodshed and sacrifices, and with guaranteed rapid victory for any technological superior aggressor is proof for disastrous failure of our teaching.

Why then is this topic almost never addressed in the math educational literature nor in textbooks for the classroom? Choose the explanation you like best:

The childish view of math: "it's not us; it's only a tool".

The priest of truth: "don't tell me who you are. I want to adore you." (E. Canetti)

The concerned math politician: "don't complicate further my well-intended math teaching".

Frontispiece from N. Tartaglia, Nova Scientia. Venice, 1537