Mathematics education in a world apart
– Where we are all together*

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The "informational society" contains contradictions, which we try to express in two paradoxes. The *paradox of inclusion* refers to the fact that current processes of globalisation, although stating a concern for inclusion, exercise (involve) an exclusion of certain social sectors. The *paradox of citizenship* alludes to the fact that education, although seeming ready to prepare for active citizenship, exercises (involves) an adaptation of the individual to the given social order. We find that much research in mathematics education ignores these two paradoxes. What could it mean, then, for mathematics education research to face the paradoxes of inclusion and citizenship? We try to indicate possible answers by pointing to perspectives, which could be emphasised more strongly in mathematics education research. These perspectives include: (1) Making the students "real", including their foreground as well as their background. (2) Humanising the teachers, which means that teachers have to be grasped as human beings and not simple as research objects. (3) Opening the curriculum, which means to open possibilities for different forms of decision-making in the educational planning process. (4) Tackling the global distribution of learning facilities, which includes a particular awareness of the (unequal) distributions of access to computers. (5) Acknowledging mathematics as part of technological actions and decision-making. In order to further support such research perspectives it becomes important to establish a *political economy of mathematics education*. This might bring research in mathematics education to deal directly with the paradoxes of the informational society.

Paradoxes of the informational society

Notions like post-modernism, reflexive modernisation, risk society, network society have all been used in an attempt to characterise aspects of our present era. Following Castells (1999), we choose to use the notion of "informational society", which helps to emphasise that the impact of technology goes far beyond industrial production and affects political, economic, social and cultural structures.

A discussion of the informational society must include considerations on globalisation as the process responsible for establishing the "world village". Globalisation refers to the fact that events in one part of the world may be

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caused by, and at the same time influence, events in others parts. Thus, our environment is continuously reconstructed in a process receiving inputs from all corners of the world. At the same time, our actions have implications for even the most remote corners of the planet.

Globalisation means inclusion, but also exclusion. According to Castells (1999), the "structural logic of the information age bears the seeds of a new, fundamental barbarism" (p. 60). Structurally irrelevant areas of the informational society constitute what Castells (1998) calls the "Fourth World". The processes of globalisation, linked to the emergence of the informational society, also include the creation of the Fourth World.

This observation draws attention to the complex dynamics of globalisation. At the same time that we are brought together by a new network of interconnections, we are also moving apart. The interplay between the global and the local is a game that connects many parts of the world in a network of flows, and simultaneously excludes regions and people from specific communities and countries. The Fourth World includes not only large regions of Africa, Latin America and Asia, but certainly also carves out large chunks of Europe, USA, Japan and Australia.

Nevertheless, the Fourth World has some relevant roles to play for the informational economy. First of all, it supplies spaces for dumping ecological problems and other side effects of industrial production. It also provides a cheap work force, which helps to ensure the material flow of goods needed in the informational economy. The globalisation linked with the informational economy seems to continue a provocative exploitation of certain parts of the world as a given.

We find that the "informational society" is a contested concept (Young, 1998). It contains contradictions, and it can develop in different directions. We shall try to summarise this fact by formulating two paradoxes, which we find to be of particular importance for mathematics education. The *paradox of inclusion* refers to the fact that the current globalisation model, which embraces universal access and inclusion as a stated principle, is also conducive to a deep exclusion of certain social sectors. Among other things, this brings about a Fourth World whose many new citizens are already to be found in the mathematics classroom. The *paradox of citizenship* alludes to the fact that the current discussions in education (including the references to the learning society) emphasise the need of relevant, meaningful education for social challenges, while, at the same time, much education appears to reduce learning to a matter of adapting the individual to social demands. On the one hand, mathematics education seems ready to prepare for active citizenship, but, on the other hand, it seems to ensure adaptation of the individual to a given social order.

Ignoring the paradoxes

How does research in mathematics education approach these two paradoxes of the informational society? A survey of research literature in mathematics education may indicate an answer. We classified research papers published between January 1999 and October 2000 in the *Journal for Research in Mathematics Education* (JRME), *Educational Studies in Mathematics* (ESM), *Recherches en Didactique des Mathématiques* (RDM), *For the Learning of Mathematics* (FLM), *Suma*, and the *International Journal of Mathematics Education in Science and Technology* (IJMEST).[1] We classified the studies according to how they deal with the context and content of learning. The context could be analysed with reference to the classroom, to the school, or to the society. The content of learning could be analysed in terms of logical, psychological, cultural, or socio-political components of what is learnt. We do not make any claim about the existence of any clear distinction between the different categories or about the adequacy of making a distinction between the context and content of learning. However, we find that the classification indicates what is taking place in research in mathematics education. The results are indicated in Figure 1.

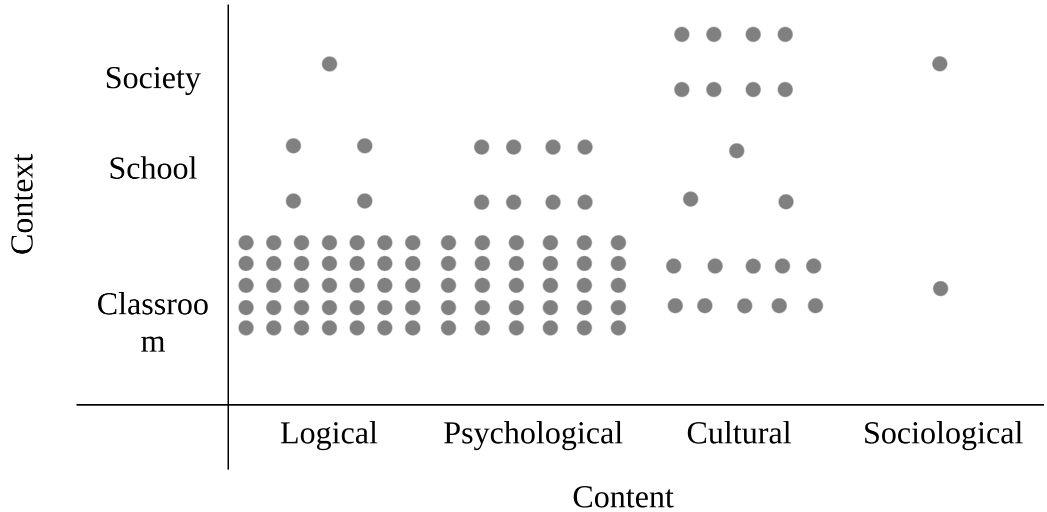

Figure 1: Indication of distribution of research papers in research journals

The figure illustrates priorities of research in mathematics education. To us it also indicates that the paradoxes of the informational society are ignored in most research. One way of doing so is to concentrate on logical and psychological aspects of the content of learning. Such a concentration could mean that the

[1] Gómez (2000) carried out a classification of the papers published in 1997 in three main international journals in mathematics education – JRME, ESM, RDM– and the proceedings of the 21st meeting of the International Group for the Psychology of Mathematics Education (PME). His classification also supports our findings.

paradox of citizenship is not under discussion. The content of learning may appear in a universal form, while the notion of citizenship must refer to the particular situation of the students. When the development of mathematical thinking is analysed independently of the overall context in which it takes place, the paradox of inclusion may also be overlooked.

Facing the paradoxes

What could it mean, then, for mathematics education research to face the paradoxes of inclusion and citizenship? We shall try to indicate possible answers to this question by pointing to some research areas that could be emphasised strongly in mathematics education research with a social and political concern.

(1) *Making the students real*. It is possible to conceptualise the mathematics learner in different ways. Research literature includes descriptions of students who seem to be eager to engage in mathematical learning. Some of these students may face difficulties, but they seem ready to face their learning difficulties and to struggle with their handicaps. In other words, students, as portrayed in research literature, do not always resemble real students. This has naturally much to do with the research perspective, which includes priorities for selecting the episodes from the classroom to be analysed. The totality of selected episodes reveals paradigmatic priorities and assumptions of the current research in mathematics education. It seems presupposed that noise in the classroom, students making obstructions, students not turning up in schools, etc. do not reveal adequate information for researching the learning of mathematics. However, we do not think this is the case. We find that it is important that students become “real” (Valero, 2002b).

Students must be grasped as human beings in a complex situation, of which the learning of mathematics is only one particular aspect. Students are not simply members of a classroom, they are part of a school and of society. Students act with reference both to their *background* and *foreground* (Skovsmose, 1994). The foreground refers to the way students interpret and conceptualise –explicitly or implicitly, consciously or unconsciously– their future, their possibilities, and their life conditions given the social, cultural, economic and political environment in which they live. The foreground frames what students do and want to do. It provides resources and reasons for the students to get involved –or not– in their learning as acting persons. Naturally, the foreground is modulated by the background of the students, which is their already socially constructed network of relationships and meanings which belongs to their personal history.

Society may assign very different foregrounds to different groups of students. Some societies, such as for example apartheid South Africa, “steal” the future of certain groups of children. This act of “stealing” destroys the foreground of some children, and in this way many possible motives for learning mathematics are eliminated. Identifying those acts of stealing is, to us, more important for understanding the learning obstacles of certain groups of children than, say, detecting certain mathematical "misconceptions". If research in mathematics education should interpret the meaning of learning mathematics for different groups of people, then the social construction of foregrounds must be considered. In this way we can hope to discuss how social processes of inclusion and exclusion determine students' processes of learning.

(2) *Humanising the teachers*. Much research literature has focussed on the mathematics teachers. An overall assumption has been that teachers' practices are determined primarily by teachers' mathematics-related beliefs and knowledge. As a consequence, "wrong" beliefs and "deficient" knowledge can be identified as the cause of "wrong" and "deficient" practices. Thus, the dominance of the school mathematics tradition is caused by the fact that teachers, explicitly or implicitly, subscribe to an interpretation of mathematics, which highlights routines and a true-false dichotomy as the basis of mathematical competence. As a consequence, belief systems and knowledge have to be changed if a change in teaching wants to be achieved.

This picture of the mathematics teacher is problematic. With reference to Skott (2000) and Valero (2002a) we want to emphasise that teachers' actions in the classroom are as complex as the actions of any human being. They are not simply determined by beliefs and knowledge concerning mathematics. Instead, teachers' actions must be understood in terms of a variety of factors such as the emergence of educational priorities, the demands of teaching-learning situations in the school, the possibilities of fruitful co-operation among teachers in the school, the particular interactions with groups of students, etc. In fact, we do not see any particular interest in searching for *explanations* of teachers' actions, but we see a great need for *understanding* teachers' actions.

Furthermore, we do not think of teachers (nor of students) as research objects in mathematics education. In case mathematics education should open possibilities for the development of citizenship, then both teachers and students have to be grasped as human beings with whom we, as researchers, may co-operate. Any process of developing citizenship as part of an educational process does not square with the conception of teachers and students as some individuals who have to be "treated" in a particular way. Such an approach only makes sense if we try to adapt students to a certain social order. But citizenship cannot be interpreted as an output of a certain educational device. Thus, the paradox of citizenship has much to do with how we conceptualise, in research, the participants in the educational process, that is, the students and the teachers.

(3) *Opening the curriculum*. Who have the possibility to participate in decision-making concerning the curriculum? Does curriculum planning and implementation open possibilities to bring into the classroom different interpretations of what relevant mathematical education practices might mean? The specification of the curriculum could take place as a top-down process or as a bottom-up process. With good reasons it can be argued that a bottom-up

strategy make it possible for both students and teachers to be included in curricular decision-making, and that this is essential for education to make part of democratic processes in society.

Nevertheless there are several issues which have to be considered when the curriculum is opened in this way. For example, it is important to consider how “local” curricula can come to operate in society. Could a particular curriculum come to constitute “second-rate” mathematics education that dooms some students to exclusion? Gorgorió and Planas (2000) discuss a research and development project intending to open possibilities for critical mathematics education with immigrant students in Catalonia. They entered in a constant struggle with interpretations of that type of education as a “soft” program that could be suitable for this particular kind of students, while educational authorities defended the need of “hard core” mathematics education programs for students expected to succeed within the educational system. Here we directly face the paradox of inclusion: A curriculum that intends to embrace issues of equity in mathematics education may risk generating yet more exclusion for the students involved.

To us this observation illustrates that the content of what is learnt in mathematics has to be discussed not only from a logical and psychological, but also from a cultural and sociological perspective, in case research in mathematics education should discuss the paradox of inclusion. The observation also emphasises how difficult it is to separate the discussion of the context from that of the content of learning. A particular content might appear relevant and interesting, when students are considered members of a classroom, but the same content might appear less attractive, when the students’ opportunities in life are considered in a broader social context.

(4) *Tackling the global distribution of learning facilities*. Much research has emphasised that Information and Communication Technologies (ICT) open and reorganise new learning possibilities (e.g., Balacheff & Kaput, 1996; Borba, 1999). We find it important, however, to consider how these learning facilities are globally distributed. Obviously, we have to do with the most unequal distribution of the ICT-facilities around the world. What does this mean for the role of mathematics education in under-resourced classrooms and schools? In particular, what does this imply for the formation of the “Fourth World”? Does the reorganisation of learning possibilities also include a reorganisation regarding inclusion in as well as exclusion from the informational society? In our view, research in mathematics education is too often set up in such a way that certain social and economic resources are taken for granted although they can be taken as such only in certain (privileged) parts of the world.

(5) *Acknowledging mathematics in action*. Mathematics is not simply a school subject, it is also a resource for technological, political, economic, military and many other forms of actions and decision-making. By means of mathematics, it is possible to establish a space of (technological) alternatives to

a present situation. Mathematics serves as a source for technological imagination, which is, however, also an imagination with lots of blind spots. By means of mathematics, we seem able to investigate particular details of not-yet-realised plans and projects. However, hypothetical reasoning about details of imagined constructions, supported by mathematics, also lays a trap since mathematics imposes a limitation on the perspectives from which hypothetical situations are investigated. Finally, being a resource for technological action and decision-making, mathematics becomes an inseparable part of our present social structures. We come to live in an environment moulded by social actors and supported by means of mathematics. In particular, the development of the informational society is closely linked to the spread of mathematics-based technologies (Skovsmose, 1999; Skovsmose & Yasukawa, 2000).

We have to consider how mathematics becomes installed in more and more technological devices (Wedege, 2000) and how it operates "behind the screen", making it possible to use mathematised tools without presupposing a deep understanding of their underlying mathematical structure, maybe even without being aware of the fact that a mathematical complexity is in operation. The implication of this is that the necessary competencies to operate with these technologies split people into two main categories: those who can operate on the surface of the technology, and those who can construct and reconstruct it. Both competencies are essential for the functioning of today's societies, and therefore it is important to discuss how mathematics education operates with regard to this split.

Mathematics education can serve as preparation for that (small) group of people who are going to develop and to maintain mathematics-based technologies in engineering, management, economy, etc. Mathematics education also serves as a preparation for that (large) group of people who are going to make use of such technologies in banking, industry, business etc. Finally, mathematics education may designate that group of people (small or large) who are not going to engage with any form of mathematics-based technologies. If research should face the paradox of inclusion, then it becomes important to examine in what way mathematics education acknowledges the different forms of mathematics in action. And in case the paradox of citizenship should be brought up for discussion, then it becomes important to ask questions like: How can "actions through mathematics" be illustrated in mathematics education? How far do we go in making mathematics education a critical activity, addressing both the wonders and the horrors of actions through mathematics?

Establishing a political economy of mathematics education

Mathematics education provides new opportunities for people; but it might also become an obstruction for certain groups to advance socially. Mathematics education presupposes resources, and we believe that it is necessary to ask how these resources – human and material – create opportunities and, more essentially, how resources and opportunities are distributed around the world. At the same time mathematics education might be recognised as an economic resource of society as it supports technological development. And these economic and political potentials of mathematics education might operate very differently in different socio-political contexts.

Ethnomathematics (D'Ambrosio, 1996; Powell & Frankenstein, 1997) challenges Eurocentrism firstly, by demonstrating that all cultures, not least traditional ones, demonstrate a deep mathematical insight, and secondly, by showing that building on this knowledge it is possible to reconstruct a mathematics education which does not recapitulate the priorities of colonisation. This has led to the formulation of ethnomathematical curricula for disadvantaged populations like, for example, the landless people in Brazil (Knijknik, 1996). However, as already indicated, this brings us back to a discussion of the formation of the Fourth World.

In order not to jump to simplified conclusions, we want to emphasise the importance of opening a new research agenda in mathematics education, which can refer to as a *political economy of mathematics education*. Such a research agenda might deal with not only the economic basis of the distribution and redistribution of learning facilities but also the whole economic basis of mathematics education around the world. Such a study might consider the economic and political role of mathematics education in the further development of the informational society, including the processes of establishing and maintaining a Fourth World. As already mentioned, it is important to make students "real" and to "humanise" teachers. Naturally, we cannot ignore the researchers, and taken as a whole this paper makes a suggestion for politicising researchers. We find that a development of a political economy of mathematics education is important in case research in mathematics education should discuss directly the paradoxes of the informational society.

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References

Balacheff, N. & Kaput, J. (1996). Computer-based learning environments in mathematics. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International Handbook of Mathematics Education* (pp. 435-468). Dordrecht: Kluwer.

Borba, M. (1999). Technologias informáticas na educação matemática e reorganização do pensamento. In M. Bicudo (Ed.), *Pesquisa em educação matemática: Concepções e perspectivas* (pp. 285-295). São Paulo: UNESP.

Castells, M. (1998): *The information age: Economy, society and sulture. Volume III, End of Millennium*. Oxford: Blackwell Publishers.

Castells, M. (1999). Flows, networks, and identities: A critical theory of the informational society. In M. Castells, R. Flecha, P. Freire, H. Giroux, D. Macedo, & P. Wilis. *Critical education in the new information age* (pp. 37-64). Lanham, Maryland: Rowman and Littefield.

D'Ambrosio, U. (1996). *Educacão matemática: Da teoria à pràtica*. Campinas, Brazil: Papirus.

Gómez, P. (2000). Investigación en educación matemática y enseñanza de las matemáticas en países en desarrollo. *Educación Matemática*. 12 (1), 93-106.

Gorgorió, N. & Planas, N. (2000). Researching multicultural classes: a collaborative approach. In J.F. Matos & M. Santos (Eds), *Mathematics Education and Society*, (pp. 265-274). Lisboa: Centro de Investigação em Educação da Faculdade de Ciências da Universidade de Lisboa.

Knijnik, G. (1996). *Exclusão e resistencia. Educacão matemática e legitimidade cultural*. Porto Alegre: Artes Médicas.

Powell, A. & Frankenstein, M. (Eds.). *Ethnomathematics. Challenging eurocentrism in mathematics education*. Albany: State University of New York Press.

Skott, J. (2000). *The images and practice of mathematics teachers*. Copenhagen: The Royal Danish School of Educational Studies, Ph.D. dissertation.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer.

Skovsmose, O. (1999). Mathematical agency and social theorising. *Preprint series of the Centre for Research in Learning Mathematics*, no. 10. Roskilde: CRLM, Danish University of Education, Roskilde University, Aalborg University.

Skovsmose, O. & Yasukava, K. (2000). Mathematics in a package: Tracking down the 'formatting power of mathematics' through a socio-mathematical excavation of PGP.). Research methodology and critical mathematics education. *Preprint series of the Centre for Research in Learning Mathematics*, no. 14. Roskilde: CRLM, Danish University of Education, Roskilde University, Aalborg University.

Valero, P. (2002a). *Reform, democracy, and mathematics education*. Copenhagen, The Danish University of Education, Ph.D. dissertation.

Valero, P. (2002b). The myth of the active learner: From cognitive to socio-political interpretations of students in mathematics classrooms. *In this volume.*

Young, M. (1998). *The curriculum of the future: From the new sociology of education to a critical theory of learning*. London: Falmer.

Wedege, T. (2000). *Matematikviden og teknologiske kompetencer hos kortuddannede voksne*. Roskilde (Denmark): Roskilde University, Ph.D. Dissertation.