

# **The myth of the active learner: From cognitive to socio-political interpretations of students in mathematics classrooms**

**Paola Valero**, The Danish University of Education, Denmark

Most mathematics education research is based on the assumption of the centrality of learners in the processes of mathematical learning. This assumption views learners as active cognitive subjects at the “centre” of the development of mathematical thinking in classrooms. This idea has constituted in mathematics education research literature what I call the *myth of the active learner*. Based on my research on mathematics education school change from a socio-political perspective, I examine this myth, discuss it and provide alternative ways of interpreting the need for placing students’ activity in the centre of mathematics teaching and learning.

## **Introduction**

I have been engaged in a theoretical study of secondary mathematics education change from a socio-political perspective. In this study I have been trying to formulate a conceptual language to address processes of change in the teaching and learning of mathematics in secondary school, according to the recent trends to reform in mathematics education. My approach to school mathematics education change has been institutional and organisational, that is, change has been conceived as a process occurring in the complex network of relations and practices among the school leaders, the group of mathematics teachers in the school, the teachers in their classroom, and the students. Furthermore, I have adopted a socio-political perspective which allows me to think about such a network in terms of the practices of people in social settings and arenas, where the political dimension of mathematics education is a central constituent of teaching and learning practices (Valero, 2002).

As a trigger to my theorisation, I carried out three case studies in three very different schools, in three different social and political contexts. In May 1998 I visited Nyspor School, a working class comprehensive primary and lower secondary school in the outskirts of Copenhagen, Denmark. In August 1998, I visited Rajas Secondary School, a middle class, predominantly Indian school in Durban, South Africa. In September 1999, I visited Esperanza Secondary School, which services working class students in a popular neighbourhood in Bogotá, Colombia. The diversity of practices, conceptions about change and presence of situations of social and political conflict in these schools constituted a rich base for my theoretical exploration. This exploration used empirical data as one of the sources of theorisation. A systematic analysis of the discourse put forwards by dominant international literature on mathematics education research has been a methodological complement to reflections triggered by the empirical

material concerning mathematics education change in the social organisation of secondary school.

In this paper I do not intend to refer broadly to my overall research. I want to concentrate on the way in which dominant<sup>1</sup> research literature in mathematics education has portrayed students. I argue that the adoption of psychological approaches to the study of mathematics education teaching and learning has allowed to build a view of students as cognitive subjects, who are expected to be active in learning as mental processes, and whose cognitive activity is central to the whole educational enterprise. This view constitutes the *myth of the active learner*. This myth is present in our formulations of the types of beings that we expect to educate mathematically. This myth, however, holds for many researchers and policy makers, but does not endure for children and youngsters for whom the learning of mathematics is one small part of the complex social experience that schooling constitutes. I argue, instead, for the need to start constructing views of the students as whole human, social beings whose motives and intentions of learning overpass the limited view of the mathematically-active, cognitive subject.

### **A turning point for reflection**

My concern for exploring the interpretations that we give in mathematics education research literature to the “centrality of students’ activity” emerged after facing the following situation:

September 1999. Bogotá. Colombia. I had already spent a week in Esperanza School and I had been observing Julia’s lessons with a 10<sup>th</sup> grade class. That day Julia was away from the school and I had agreed with her to take over her class. As a visitor in the school I was the only one who had time available for covering her up. This group of around forty students were working on trigonometric identities. I was supposed to give them the exercise worksheet that they had started to solve the previous session. While many in the class worked, two male students engaged me in a chat. These two boys were supposed to be doing the mathematics. Instead, they looked at the worksheet and laughed, looked at me and laughed, looked at their mates and laughed. They called me for help, but in reality they were curious to know about my intentions and motivations to be in their school. They wanted to know about my life, where I lived, where I had studied, why I was living in Denmark and not in Colombia. They could not understand why I was there in that “poor” school, talking to poor people if:

Andrés: We can see in your face that you have never suffered. You’ve got it easy.

I was sincere with them. I have not suffered, that is true. But that did not mean that I had got it easy. I studied hard to have the chance to do what I was doing. My intention was to tell them that there were reasons to study and to be interested

---

<sup>1</sup> The adjective “dominant” here refers to what is most represented in mathematics education research. As Skovsmose and Valero (in press) show “dominant” research in the field adopts mathematical and psychological approaches, and tackles analysis at an individual or classroom level.

in the school and maybe in this mathematics. But they could not see it in their lives:

José: The only class I would like to pay attention to is English because I want to get out of this fucking country and go to the US. Still I don't even manage to learn how to say "Yes, good morning"

This conversation shook me. These two boys clearly drew a line between them, as members of the school and of a subculture, and me, as an outsider not only to the school but also to their life experience. I interpreted their words "*We can see in your face that you have never suffered; you've got it easy*" as a marking of me as belonging to a "world" that differed from that of people who have to struggle, among others because of having limited economic resources. This conversation also made me question whether my intention of making them feel interested about the studies they were doing, especially in mathematics, was meaningful to them. Probably they could not see any reason to study given the limited possibilities in their future, particularly in a moment of deep economic crises such as the one that Colombia lived at the end of the 1990's.

I take the situation of two particular students in Colombia as an exemplar of the potential reactions of many students around the world. During my stay in Rajas Secondary School I did not come that close to students. However I suspect that in Rajas, as well as in many less privileged schools in South Africa, a researcher could meet similar reactions. What do these students have to tell us? This episode represented an opportunity to examine the myth of the active learner.

### **The myth of the active learner**

A myth can be defined among others as "an unproved or false collective belief that is used to justify a social institution" (Webster's Dictionary, p. 1272). I argue that dominant research literature in mathematics education associated to current reform trends have built a conception of the child, the mathematical learner, that is used to justify particular views of what mathematics education in school is about. This conception, nevertheless, may be taken as "unproved or false" in the sense that does not give a full picture of the beings it intends to talk about. In that sense that portrays almost mythological beings, instead of the concrete children that we meet in classrooms. What is this myth of the active learner about and how has it been constructed?

One of the main characteristics of the current reform discourse is the placement of students and their mathematical thinking development as the target of the whole educational enterprise. The reform discourse, as espoused in different documents and research articles dealing with mathematics education, portrays students mainly as *cognitive subjects* around which all learning and for whom all teaching happens. There are different reasons for this. Reform ideas adopted mainly Piaget's genetic epistemology and more recently Vygotsky's socio-cultural epistemology as the theories of learning that more appropriately

give an account of human learning (Skott, 2000). Numerous researchers have been engaged in studying the work of these two psychologists in order to recontextualise their theories into mathematics education (e.g., Steffe et al., 1996). Independently of which psychological developmental theory is chosen, researchers who have been involved in rethinking mathematics learning from (socio)constructivist and socio-cultural perspectives have placed an emphasis on the student as a significant unit around which learning happens. Either as a result of mental processes of assimilation and accommodation, or of internalisation and symbolic mediation, the cognitive subject is the centre of learning. The way in which students come to develop mathematical thinking and meaning is the central aim of the whole reform discourse.

From a review of different trends that tackle diverse aspects of students' learning, we can conclude that the dominant reform discourse portrays students as universal cognitive subjects. The adherence to developmental psychology has meant, on the one hand, the adoption of a universal, individual child, representative of human race, whose cognitive development can be described in terms of standard mental processes. Mathematics education research talks about this universal, normal child and how he thinks mathematically. Although this notion of universal child seems to have been challenged by socio-cultural approaches based on Vygotsky's and his followers' theories –which emphasise the social and cultural nature of the tools that mediate learning– I find that the fundamental assumption of the importance on focusing on the cognitive subject has remained untouched. Instead studies adopting this approach complement “psychological constructivist analyses of individual children's mathematical activity” (Cobb et al., 1996, p. 7). Although they add complexity to the view of the student as a cognitive subject, they are not changing in essence the idea that what interests for mathematics education and its change is students' mathematical cognition.

There are different expectations about how this cognitive subject has to act in relation to his cognitive development. Many chapters in Bishop et al. (1996) put forward a discourse which describes students as autonomous and responsible for their own mathematical learning, problem solvers, aware of their thinking processes –that is, “meta-reflectors”–, technologically literate, mathematical communicators, mathematically confident, and able to connect mathematics in the classroom with their reality. From this is clear that students are expected to be mathematical competent actors.

Besides the subject as a cognitive agent who acts in relation to mathematics, the discourse of reform also portrays students whose main interest is mathematical learning and whose intentions are actually placed in engaging in that learning. Any other aspect, interest or intention is underplayed and in fact is not mentioned as a part of the dominant discourse. This can be seen in the fact that other aspects of the human being make part of the discourse only if they are strongly connected with mathematical learning. Thus, cognitive agents –besides

their basic thinking capacities– have likes and dislikes, beliefs, feelings, gender and class, and a social context to the extent that these aspects relate with the capacity of the person to think and act mathematically.

In summary, the view of the student portrayed in the reform discourse espoused in dominant research in mathematics education is that of mainly individual cognitive subjects who think and act mathematically, and whose any other dimensions except from the cognitive are anyway associated to the process of learning mathematics. This view is justified, from the point of view of the research and of practice on the assumption that what interests to change in mathematics education is the thinking processes involved in the learning of mathematics.

### **Re-humanising the student**

It is understandable that mathematics education research and the reform discourse had constructed such a view of students. As professional mathematics education researchers we tend to prioritise those dimensions of the human being that we see as more directly under our area of study and influence. Nevertheless, I find that the risk of that narrow focus is high since it might not allow us to understand the whole complexity of what learning mathematics is, not only from our perspective as researchers, but specially from the perspective of the learners and their perception of their experience. From their perspective, probably, the myth of the active learner falls apart.

If I had to represent the “reform student”, I would be forced to draw a being that would look like a sci-fi being, an outer-space visitor, with a big head, probably a bit of heart, and a bit of body. That being would be mainly alone and mostly talk about mathematics and its learning, and would see the world through his school mathematical experience. It would be a “schizo-being” since he has a clearly divided self: that one that has to do with mathematics, and the one that has to do with other unrelated things. Of course, those other unrelated things are secondary. This is the child that many researchers in mathematics education have in mind in their work in practice or theory.

But I have not seen any of these “weird beings” in any of the classrooms I observed in Denmark, South Africa and Colombia. Is it true that students are “schizo-mathematics-learners”? I mean, is it true that we can conceive students as beings who are mainly learning mathematics? Is it true that they act in one particular form? Is it true that they are interested in learning mathematics? Is it true that they can separate their experience in learning mathematics from their whole school experience and from their whole life experience? The words of the Colombian students “*The only class I would like to pay attention to is English because I want to get out of this fucking country and go to the US. Still I don’t even manage to learn how to say ‘Yes, good morning’*” made me think that students are not only mathematics learners, are not necessarily interested in learning mathematics, do not act in the ways we expect them to do, do not

perceive a separation between their mathematical learning experience and their whole school and life experience, and have a whole human integrity.

My argument contends that the view of students as “schizo-mathematics-learners” appears to be limited when questioned from the students’ point of view. If we try to incorporate the students’ point of view, we may find that we need to broaden our perception about the students and their school mathematical experience. There are at least two salient issues concerning learners that pop up from a socio-political analysis.

### *Diversity of reasons for learning*

The reform discourse has constructed the image that students are intrinsically interested in being part of classroom activities and have the intention of learning mathematics. Many research papers that describe classroom situations depict students performing mathematical activity. Classrooms are organised: Students behave well, and teachers never have to deal with discipline problems, lack of attention or insufficient motivation. These descriptions lead to a very unproblematic representation of the students, their engagement and interest in mathematics learning. The “clean” classroom of research does not fit with the diversity that one finds in real classrooms. Certainly it did not fit with the environment in which Andrés and José lived, where among the forty students, there were some who participated, some who remained quiet, and some who did not care about the class and were talking about other things but the trigonometry problems they were assigned. I observed the class for some weeks when Julia, the teacher, developed the topic of geometrical identities. In order to provide possibilities for students’ having a richer understanding of the topic, she implemented a sequence where graphic and symbolic representations of the identities were combined to show the equivalence of, for example,  $\operatorname{cosec}^2 \alpha$  to  $\cot^2 \alpha + 1$ , in the case of the identity  $\operatorname{cosec}^2 \alpha = \cot^2 \alpha + 1$ . In that sequence of class there were only few who contributed to the discussion. Andrés and José were among the silent majority who hardly ever participated. Sitting among the students, I noticed that students did different things while the class was going on. Chatting about the football match or the last episode of the most popular soap opera was a parallel activity going among the students in the classroom. This diversity in student’s participation in that particular classroom is not unique<sup>2</sup>.

Mellin-Olsen (1987) centred part of his social pedagogy of mathematics on students’ possibilities for action in relation to school mathematics. He explored what might be at the base of students’ activity, from the point of view of the students. At the start of the time when the core ideas of the constructivist reform were being formed, Mellin-Olsen had already pointed out to an important issue: “[A]s educators we see them [students] decide to learn or not to learn. As

---

<sup>2</sup> For mathematics classroom descriptions showing this diversity, see Vithal (2000).

educators we are cheating ourselves if we do not make this phenomenon central to our theorisations.” (p. 157) Students may decide not to learn. This fact opposes the view of the student that dominates in the reform discourse. Mellin-Olsen paid attention to the fact that students’ activity should not only be described in cognitive terms, but mainly in sociological terms. The activity of a person –even if it is mathematical– is the result of the person’s interaction with her environment. That environment is a field where the person meets other people and social institutions. There, the person both expresses her own ideological constructions about how to act in the environment, and is exposed to the expression of other’s ideologies and of broader social ideologies. From these interactions emerge rationales for behaviour, which are belonging to the individual but constructed in the field of social interaction. The rationales for behaviour that drive school learning may be of two kinds: *instrumental* and *social*. The instrumental rationale is related to the “school’s influence on the future of the pupil, by the formal qualifications it can contribute [...] In its purest form the I-rationale will tell the pupil that he has to learn, because it will pay out in terms of marks, exams, certificates and so forth” (p. 157). The social rationale relates to the perceived value of the knowledge learned beyond the school, that is, for example, the importance of mathematical knowledge outside of school. These two rationales provide justifications for individual activity: “knowledge is regarded important by the pupil for external examinations and for itself.” (p. 158)

Mellin-Olsen’s contribution concerning the view of the student is precisely the legitimisation of the need to consider a broad understanding of the motives for students’ learning. The existence of strong instrumental rationale for students’ participation in a mathematics classroom is a strong motive for action in many societies where school mathematics acts as a gatekeeper to further studies. Many students privilege the functional rationale for getting engaged in mathematics learning because of the possibilities that it represents in terms of qualifications and improved possibilities of academic and labour market access. Just as an example think about many South African students in relation to the Matric Exam...

There are reasons for students to perceive in a broader way their participation in mathematical learning. Mellin-Olsen’s functional rationale offers the possibility of, first, recognising that there are students who decide not to learn, and, second, integrating functional reasons for learning as legitimate and coexisting rationales that cannot be separated from those focusing on the mathematical content. Mellin-Olsen’s also allows pointing out to the sociological –and not only cognitive– character of motives for learning.

#### *More than classroom bounded beings*

This point of view also allows us to allude to another of the characteristics of the dominant reform discourse about students. Students are mainly depicted as

context-free human beings. We could argue that most of the research produced during the 1990's adopted views of context restricted to *task context* –the “real” or mathematical formulation of a mathematical problem– and to *interaction context* –the context of social interactions among teacher, students and mathematics in the classroom. Let us examine the implications of these views of context on the issue of the representation of students as context-free beings.

In radical constructivist research, where the notion of context is mainly associated to the task-context, students are not taken to be concrete human beings in a particular social and historical situation, but beings representing an abstract “human being”. Through the study of one, some or many children, we can generate understanding of the thinking processes of the universal child. The emphasis on describing the mental processes that these “abstract students” follow have the effect of detaching children from the frames in which concrete human beings exist (e.g., Maher & Martino, 1996).

Research in the more social-constructivist and interactionist trend, besides characterisations of the type presented above, give a bit more of context to children. The context offered in this case is the classroom context. Children stop being laboratory universal beings, and get reified as “classroom bounded beings”. Students’ mathematical learning is situated with respect to the classroom microculture. In this microculture, individual interpretations and reasoning emerge simultaneously with the development of the classroom’s social norms, its sociomathematical norms, and its mathematical practices (e.g., Cobb, 2000).

The influence of socio-cultural studies of learning from anthropological perspectives put forward the idea of context in mathematics education as the *situation context* –the social setting where learning activity takes place. These studies helped opening the view of context and allowed researchers to portray students as more than classroom bounded beings. Many of the researchers who claim to have followed this trend have redefined the possible meaning of students’ context in terms of the settings for mathematical activity. Mathematics learning can be seen not only in terms of the cognitive priority of the discipline, but as a process in which individuals “learn to *be*” (e.g. Boaler & Greeno, 2000). Students in this kind of discourse seem to be human beings within the context of a restricted community. Despite of the enlargement of the notion of the students’ context here, it seems that the elements of the socio-political context of the students, as fully historical members of a large society, dilute when participating in a *mathematics* community of practice.

The discourse of the reform about the student, as portrayed in dominant research, depicts a student who is either a context-free universal mind, a classroom bounded being or a participant of a limited learning community. This view opposes the nature of the students that one meets in real classrooms. The case of Andrés and José in Esperanza Secondary School illustrates two youngsters who are fully social beings, in a particular historical time,

geographical location, and social position. The characteristics of their *socio-political context* may not be discarded when considering what a particular mathematical learning experience means for them. The idea behind this postulate is that understanding the real social, historical, political and economic situatedness of students' intentions of participation in mathematical learning implies keeping a strong and clear connection between the students' microcontext –the task, the interaction and the situation context– and their broad socio-political macrocontext as member of a particular society in a given historical time.

Andrés' and José' s words in our conversation emerged in the latter type of context. It is probable to interpret them as an indication of the connection that for them exists between their school experience and their larger social, historical and political context. Does it make sense to study in a place where there seems to be no future? That is probably the question revolving around in the heads of some of Esperanza School students –or of some South African students in a township, or “second generation immigrant” students in a Turkish ghetto in Denmark, or a Palestinian student in the middle of the cross-fire... Mellin-Olsen (1987) proposed his notion of the instrumental rationale of learning in a context in which it was evident that gaining qualifications represented a possibility for the future. But what if the “future” becomes unclear from the students' point of view?

Skovsmose (1994) introduced the concept of *intentionality* as a central notion in any critical philosophy of mathematics education. He formulated the issue of the rationales for action in terms of the learner's intentions of learning. Without a conscious will for engaging in action, there is no learning. A person's intentions for acting are connected with the person's *dispositions*. Skovsmose refers to dispositions in terms of the person's *background* and *foreground*. The background, he says, “can be interpreted as that socially constructed network of relationships and meanings which belong to the history of the person” (p. 179). The foreground refers to “the possibilities which the social situation makes available for the individual to perceive as his or her possibilities” (p. 179).

There are two remarks about Skovsmose's notions. First, students' background and foreground are not connected linearly by a cause-effect relationship. In other words, the foreground is not the result of the background. There is no determinism between the two. Second, the kind of relationship established between background and foreground as sources of learning intentions depend on how the individual interprets both his personal history and his potential future in relation to a particular social situation. In the case of the Colombian students, this means that their decision to engage in schooling and in school mathematics is not determined by their belonging to a particular social class. Neither does it mean that their choices in relation to what they could gain from learning mathematics in a future is determined by their present social and economic position. This means, instead, that students constantly weigh the

choice of participating in schooling and school mathematics learning taking into consideration their personal history and the possible future in a whole social context. Sometimes the combination background–foreground–context is such that it may make sense for some to engage in mathematics education as a means of acquiring socially valued qualifications that are needed to secure a better future position –as in the case of some of the students in Rajas Secondary School who insist in getting a place in a higher grade mathematics class. But some other times, the combination shows a less optimistic landscape in which no engagement will any way improve the chances for the future, as may be the case of Andrés and José in Esperanza School. The strong influence of the broad historical, social, political and economic context in which students live and generate intentions for learning cannot be ignored in mathematics education.

### **Coming closer to a socio-political perspective on the students**

I started questioning the ways in which the reform discourse, represented in research literature, considers the students to be “central”. The dominant portrayal of students as cognitive subjects, involved in the learning of mathematics and contextualised in the classroom, which is at the core of the myth of the active learner, was challenged. Instead, I proposed a “re-humanised” view of students as whole learners, who have multiple motives for learning, and who live in a broad context which influences their intentions to participate in school mathematics practices. Bringing students to the centre means considering students as the beings that I have just described, and not as the “schizo-mathematics-learners” that the reform discourse proposes. Students are important in mathematics education because they are full human beings to whom the experience of engaging in mathematical learning has to make a broad meaning. That meaning, which is at the base of acting for learning, is related to multiple motives, rationales and intentions emerging from the continuous interplay between the students’ background, foreground and context.

Conceiving students in the way that has been proposed here is an essential characteristic of a socio-political perspective in mathematics education. There are several reasons for this. First of all, having in mind students who are full human beings, and not only cognitive subjects, allows us to recognise the *agency* that students have in the whole educational enterprise. Saying that students are agents does not mean that they construct knowledge as a mental process –“cognitive agents”– but that they act in complex social situations –“sociological agents”. As we saw in the previous discussions, students build different reasons to get involved in school mathematics practices. Their participation in those practices, which are perceived by the student as a whole social experience and not only as an intellectual, mental or cognitive endeavour, stems from their intention to act and influence the social space where the learning and teaching of mathematics takes place. Students are participants of a

social situation, and the development of that social situation depends strongly on the agency that they can exercise in it.

Second, the recognition of students as agents places students in a different position of power in relation to the teacher and how school mathematics practices develop, at least in the classroom, as a result of this balance. In a real classroom situation, there are students who seem to follow the didactic game set in place by the teacher. Those are the students whose intentions seem to “be aligned” with some of the teacher’s intentions in a situation. These are the well behaved, bright students that many research reports show. In this situations the teacher has the lead and the responsibility for the “accountability” of mathematics teaching and learning. But this type of students tends to be, in real classrooms, only a small portion. There are others whose intentions diverge from the teacher’s. They decide to participate in different ways: keeping silent, bullying, or resisting. These are groups of students who openly adopt an attitude of no-participation in the game proposed by the teacher. From a traditional mathematics education viewpoint, these would be considered “deviant” or “problematic” students who need to be treated in order to restore the normality of the classroom. From a socio-political outlook, these students are expressing their agency. Instead of trying to normalise them, both research and practice need to consider seriously the intentionality behind their participation. Recognising resistance and its motivation from the students’ viewpoint is the base for a negotiation between teacher and students about how to make evolve school mathematics practices. Even in the case of the “aligned” students the same exploration and negotiation is needed. The imposition of the teacher’s agenda on students’ intentions risks generating the situation of not meeting students in what they perceive as being relevant about their school mathematics experience. This gap and its maintenance could explain the tendency towards reform ineffectiveness. Mathematics education without open *negotiation of intentionality* risks being a failure.

Third, if students are agents and negotiation can help bringing their intentions into the educational scene, real *empowerment* may take place. Empowerment in the reform discourse has been presented as the capacity that students may get by acquiring the intrinsic power of mathematics. It is the knowing of something –mathematics– what confers power to students. This view of power internally founded in mathematics is problematic (Skovsmose & Valero, 2001; in press). In the situation in which students are recognised as agents of the educational process, empowerment does not emerge from the “possession” of mathematics, but from the position that students adopt to influence the social practices where mathematics are taught and learned. Empowerment, then, is not passed from the teacher to the student by means of the transference of a “powerful knowledge”. As socio-cultural studies on learning have shown, knowledge may not be transferable (e.g., Lave, 1988). Sociologists and micro-political analysts have also argued that power is not an

intrinsic characteristic of a person or a thing, but the manifestation of a relation in which people position themselves in order to influence the outcomes of a situation using diverse tools (Foucault, 1972). If neither power nor knowledge can be transferred, then empowerment needs to be defined in terms of the potentialities for students to participate in school mathematics practices. They get empowered when, through that participation, they position themselves in ways that are significant for the development of the practice. In that positioning intervene their intentions, their negotiation with the teacher about them, and their actual involvement in actions connected to mathematics. The “learning” about positioning themselves in diverse practices and using diverse resources that schooling offers to them is what constitutes empowerment in a school situation.

Finally, agency, negotiation of intentionality and empowerment are essential characteristics of *students as political actors* in the (mathematical) educational scene. The term “political actors” here means that power is gained by acting in a social situation. Politics, then, is not only what politicians do when invested with the authority of the constituency who elected them, or what citizens do when electing their representative politicians. Politics is what people do everyday when interacting to collectively produce and transform their material living conditions (Valero, 1999). This idea of political action allows us to think of mathematics education in social and political terms. A socio-political approach in mathematics education conceives students as real actors –and not receivers. It is in these terms that it makes sense to consider students a part of mathematics education from a socio-political perspective.

## References

- Bishop, A., Clements, K., Keitel, C., Kilpatrick, J., & Laborde, C. (Eds.). *International Handbook of Mathematics Education*. Dordrecht: Kluwer.
- Boaler, J. & Greeno, J. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171-200). Westport: Ablex Publishing.
- Cobb, P. (2000). The importance of a situated view of learning to the design of research and instruction. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 45-82). Westport: Ablex Publishing.
- Cobb, P., Jaworski, B., & Presmeg, N. (1996). Emergent and sociocultural views of mathematical activity. In L. Steffe, P. Neshier, P. Cobb, G. Goldin, & B. Greer (1996). *Theories of mathematical learning* (pp. 3-19). Mahwah, N.J.: Lawrence Erlbaum.
- Foucault, M. (1972). *The archaeology of knowledge*. New York: Pantheon.
- Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge: Cambridge University Press.
- Maher, C.A., & Martino, A. (1996). Young children invent methods of proof: The gang of four. In L. Steffe, P. Neshier, P. Cobb, G. Goldin, & B. Greer (1996). *Theories of mathematical learning* (pp. 431-447). Mahwah, N.J.: Lawrence Erlbaum.
- Mellin-Olsen, S. (1987). *The politics of mathematics education*. Dordrecht: Kluwer.
- Skott, J. (2000). *The images and practice of mathematics teachers*. Copenhagen, The Royal Danish School of Educational Studies, Ph.D. dissertation.

- Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer Academic Publishers.
- Skovsmose, O., Valero, P. (2001). Breaking political neutrality. The critical engagement of mathematics education with democracy. In B. Atweh, H. Forgasz, B. Nebres (Eds.), *Socio-cultural aspects of mathematics education: An international research perspective* (pp. 37-56) Mahwah, NJ: Lawrence Erlbaum.
- Skovsmose, O., & Valero, P. (In press). Democratic access to powerful mathematical ideas. In L. D. English (Ed.), *Handbook of international research in mathematics education: Directions for the 21st century*. Mahwah, NJ: Lawrence Erlbaum.
- Steffe, L., Neshet, P., Cobb, P., Goldin, G., & Greer, B. (1996). *Theories of mathematical learning*. Mahwah, N.J.: Lawrence Erlbaum.
- Valero, P. (2002). *Reform, democracy, and secondary school mathematics*. Copenhagen, The Danish University of Education, Ph.D. dissertation.
- Valero, P. (1999). Deliberative mathematics education for social democratization in Latin America. *Zentralblatt für Didaktik der Mathematik*, 98(6), 20-26.
- Vithal, R. (2000). *In search of a pedagogy of conflict and dialogue for mathematics education*. Aalborg (Denmark), Aalborg University, Ph.D. Dissertation.
- Webster's Encyclopedic Unabridged Dictionary (1996). New York: Gramercy Books.