Epistemology, methodology and the building of meaning in a community of practice in England 1960 – 1980

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This paper describes the paradigm shift in attitudes to the teaching and learning of mathematics in English schools from the publication of the Mathematical Associations's 'Report on Mathematics in Secondary Schools' (1959), to the Association of Teachers of Mathematics' 'Notes on Mathematics for Children' (1977). The epistemological claims of the new paradigm are demonstrated, and the philosophical basis is examined. Some links are made with recent developments in cognitive theory research.

Introduction

In earlier papers (Rogers, 1999, 2000) I explored social and political aspects of the development of the mathematics curriculum and the associated pedagogical practices. Here I recall some events of the more recent past in the history of the English mathematics curriculum. This account focusses on a time of great change in England, not only for the mathematics curriculum, but also for the education system in general. The full complexity of this change can only be hinted at here, and my choice of events and documents is necessarily limited. I also admit that I have been an active participant in some of the enterprises I relate and my views are therefore influenced by this participation. Since the historian's enterprise is one of interpretation, which has an ideological background resting in a set of beliefs which are necessarily contemporaneous to the writer, and which determine the questions and the ways in which they are asked, I appeal to those colleagues who have shared some of these experiences to compare their interpretations with my own. Historiography has a narrative character, and I must recognise that I have been a participant and am now an observer reflecting on part of the fabric I am describing.

This paper is an attempt to examine more closely some of the evidence and circumstances surrounding the development of a particular style of practice in mathematics teaching in England from the early 1960s to the mid 1970s. This period saw great expansion in the Teacher Training Colleges, and the development of 'teacher training' into 'teacher education' with the inception of the Bachelor of Education degree as a means of raising the academic status of teachers,[1] to achieve the ideal of an all-graduate teaching profession. At the beginning of this period, there were examinations at age 11 to select pupils for Grammar Schools, where they were expected to continue to university, while those who 'failed' were confined to Secondary Modern and Technical schools,

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[1] As still happens in many countries, there is a social and academic divide between Primary teachers who attended 'training' course and Secondary teachers who are usually university graduates in their subject.

P. Valero & O. Skovsmose (2002) (Eds.). Proceedings of the 3rd International MES Conference. Copenhagen: Centre for Research in Learning Mathematics, pp. 1-9.

to provide the majority of the work force.[2] However, the Labour government of the time began to establish the Comprehensive schools, thus bringing the grammar and secondary modern pupils together to be taught (ideally) in mixed classes. In concert with this change, the Schools Council,[3] the Nuffield Foundation,[4] and Local Education Authorities were actively encouraging ‘grass roots’ curriculum development. As far as the mathematics curriculum was concerned, the social changes challenged many preconceptions about content, teaching methods, and expectations of pupils’ achievement. At the same time the ‘Modern Mathematics’ movement was a significant force for curriculum change and was interpreted and developed by various groups in different countries in quite different ways. A considerable amount of material appeared in England from the United States, and a particular influence from Europe was the “Royaumont Seminar” which encouraged the modernisation of the mathematics curriculum for largely economic reasons (OEEC 1961a, 1961b). In this paper I will confine my remarks to some of the circumstances surrounding a significant paradigm shift in pedagogical practice, which concerns the early days of the Association of Teachers of Mathematics.

The dominant paradigm

The dominant paradigm for educational practice was founded on the belief that some were born to lead, and others to follow. This was a result of the ‘ideological and pedagogical divide’ I described earlier (Rogers, 1999).

Consequently, for mathematics, very different syllabuses, expectations, and pedagogical practices were to be found in the grammar and secondary modern schools. This paradigm was transmitted and reinforced in the various reports published by the Mathematical Association.[5] For the purpose of this account I begin with their report on the *Teaching of Mathematics in Primary Schools*, first published in 1956 and reprinted without alterations until 1966. A number of committees had met to study the problem, but failed to agree, and by the time the final committee was appointed in 1950 a new doctrine had emerged, which was embodied in the 1956 report.[6] Even today, we might not argue with the broad statement of principle:

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[2] This is the effect of the earlier social ‘divide’ that I addressed in my paper of 1998 (see MEAS 1)

[3] The Schools Council was a quasi-autonomous organisation funded by government, and set up to encourage and support curriculum development.

[4] The Nuffield Foundation is a Charity which supports Educational Research and Development

[5] For a detailed account of the history of the Mathematical Association, its social context and its dealings with other organisations, see Price, 1994.

[6] Some time earlier, Caleb Gattegno was co - opted onto this committee and continued on the new committee appointed in 1950 until the report was published. See section 3 below.

> “...... children, developing at their own individual rates, learn through their active response to the experience that comes to them; through constructive play, experiment and discussion children become aware of relationships and develop mental structures which are mathematical in form and are in fact the only sound basis of mathematical techniques. The aim of primary teaching, ... is the laying of this foundation of mathematical thinking about the numerical and spatial aspects of the objects and activities which children of this age encounter .... this report is concerned with an approach to the relations and ideas which are fundamental not only to the art of reckoning, but to Mathematics as a whole. ....” (M.A. 1956 pp v/vi)

It is interesting to note that with reference to the justification of the claims made in the introduction, the bibliography mostly contains references to psychology and research from 1895 through to 1946. The exception is Piaget’s *Child’s Conception of Number* (1952), and one can speculate that this had a particular influence on the writers of the report.

A report on *Mathematics in Secondary Modern Schools* was produced in 1959 which was intended to be read in conjunction with, and as a continuation of the earlier Primary report. Here the tone is quite different, in the statement of terms of reference we find:

> “The mathematical ideas and teaching methods here discussed are those which we consider suitable for the great bulk of secondary school population between 11 and 15 years of age, who have shown no early signs of readiness for Mathematics as an abstract study, or at least have not achieved the attainments in arithmetic traditionally associated with the ablest of their age group... Its use as a specialised tool may be needed by some later in life, but readiness for many mathematical techniques comes with an intellectual maturity only reached by most modern school pupils some time after entry, if at all.” (M.A 1959 pp 1/2)

Clearly, expectations for these pupils are limited, ‘intellectual maturity’ is the criterion, and the attitude is one of a ‘vocational’ rather than ‘professional’ future for the pupils where few, if any, are expected to go on to any form of further education. *A Second Report on the Teaching of Arithmetic in Schools* was published in 1964 which discusses the teaching of arithmetic as part of a general mathematics course, from the end of the primary school to age 16. This is intended;

> ”... for teachers of boys and girls of good ability such as are found in grammar schools and in the grammar streams of comprehensive schools. There are many other pupils who are still not ready at 11-plus to begin such a course as is here envisaged and appropriate guidance for their teaching may be found in ... *Mathematics in secondary modern schools*.” (M.A. 1964 p 1)

By now, the modern mathematics movement was well under way, and so arithmetic:

> “...should be taught in such a way that it leads on to the rest of mathematics, not only algebra and trigonometry, coordinate geometry and calculus, but also the new topics now being introduced into school syllabuses...” (M.A. 1964 p 3)

However, apart from statistics (which was not really a new topic), hardly any mention is made of any ‘new’ mathematics. Towards the end of the book, we find some suggestions for classroom procedure:

> “The teacher has ..... two lines of approach. The first is traditional and familiar: it is teacher-centred and relies on exposition and examples worked on the blackboard, but the pupil learns largely by listening. In the second the emphasis is on the pupil’s activity: the teacher’s task is to contrive the appropriate stimulus to learning, sometimes by posing a challenge or problems and guiding the pupil to discover a solution, at other times by suggesting and activity .... leading to the discovery of a generalisation. ” (M.A. 1964 p 84)

Apart from this brief hint at ‘guided discovery’ the suggestions only concern organisation, choice of examples, setting out, marking, etc., but provide no discussion of the actual pedagogical challenges produced by the new educational system. Clearly, the problems of teaching pupils in mixed classes are being felt, but it was difficult to break out of the established pattern. Some reverted to the old pattern, by creating different ‘ability sets’ in the comprehensive school.

Establishing a new community of practice

Mathematics, like any other human enterprise develops in ‘communities of practice’ (Adler, 2000), where newcomers are initiated into the practices which are passed on from one generation to another. Not all of the meanings or variations of practice are written down, but are shared as ‘tacit knowledge’ by the community (Polanyi 1964). Before the late 1950s, the Mathematical Association was the principal authority whereby the beliefs and practices about teaching were handed on. The members were almost exclusively university graduates who had little or no knowledge of psychology and usually no pedagogical training, and so the broad consensus of expectation and action was driven by their training as mathematicians. This is not to deny that the members had high expectations for their pupils, but these pupils were the privileged few who were found in grammar and private schools. At this time, the teachers in the secondary modern schools were mostly those who had attended a two year course in a training college where the mathematics was mostly limited to that expected of secondary pupils.

It became clear that some teachers in the secondary modern schools were very concerned with the problems of teaching their pupils, and as new entrants to the profession began to read the work of Piaget and realise that the problems could be seen from different points of view, the underlying epistemological beliefs that guided the established practices were seriously challenged.

Breaking the mould: the beginnings of ATM

The ATM was founded as the Association for Teaching Aids in Mathematics (ATAM) in the mid 1950s by Caleb Gattegno, who had been a member of the Mathematical Association committee which produced the Primary mathematics report. There had been a clear ideological split between the established guardians of practice and the radical ideas represented by Gattegno and his sympathisers. The wider implications of emerging psychological theory meant that the assumptions about teaching mathematics at all levels had to be reexamined. The founder members of ATAM were initially concerned with the plight of pupils in primary and secondary schools whose mathematical diet was extremely limited, and they placed great emphasis on the development and use of a range of apparatus to assist pupils learning.

ATAM produced a quarterly journal together with a considerable number of pamphlets for teachers, explaining mathematical topics, reviewing books and apparatus, discussing psychological theory and sharing experiences from the classroom. Many speakers were invited from Europe and the USA to address the annual conference, and their accounts of projects, experiments and classroom experiences were published in the journal. In 1962 the organisation changed its name to the Association of Teachers of Mathematics to widen its appeal and show it was concerned with the mathematical education of students at all levels, from primary school to university.

In 1962 twenty ATM members came together for a ‘writing week’. Group writing became a particular feature of many ATM publications. Individual drafts were critically reviewed by the group, and redrafted, or combined, before they were finally accepted. This process clearly made many difficult personal and intellectual demands on those involved.

The product of this first group writing was *Some Lessons in Mathematics* (ATM, 1964) which contained many suggestions for the secondary classroom, some ‘modern’ topics and some new approaches to well estsablished curriculum topics. The tone of the book was provocative; posing questions for teachers not only about the content of the curriculum, but also about the relevance of the mathematics, the teaching methods, and motivations for both teachers and pupils. The Introduction states:

> “The lessons which follow show a variety of styles and approach, but it seems to us that behind many there is a common strategy. Mathematics does not start from a finished theorem in the textbooks; it starts from situations. Before the first results are achieved there is a period of discovery, creation, error, discarding and accepting. This period is notoriously difficult to discuss, or even to describe in a convincing way, but in this book we have tried to make this period our concern.” (ATM 1964, p.2)

The nature of a ‘mathematical situation’ became a focus for debate, and the case for reform rested on a desire to produce classroom environments in which as many pupils as possible worked creatively. It was therefore the teacher’s responsibility to understand new knowledge and use it as a basis for a new ‘technology of teaching’.

The idea of a period of experimentation before a formal stage was reached, became a hallmark of the ‘ATM approach’, and the use of all kinds apparatus from simple everyday things like tracing paper or mirrors, to specially made ‘structural material’ and films were seen as essential to the exploratory period. Some time about 1966/67, copies of *Proofs and Refutations* (Lakatos 1963 - 64) began to circulate among ATM members, and in Lakatos’ work, many found the philosophical justification for the methodology of the experimental approach, and the social context for evaluation of the mathematics produced.

Another group writing project in 1965 produced *Notes on Mathematics in Primary Schools*, published in 1967. This was largely based on observations of children learning mathematics, and demanded that teachers should themselves engage with at least some of the mathematics discussed. The Introduction to this book contains the following ‘statement of belief’:

> “Mathematics is the creation of human minds. A new piece of mathematics can be fashioned to do a job in the same way that , say, a new building can be designed .... The invention of new algebras and new geometries ... have shown that mathematics cannot be an absolute, given *a priori*, or a science built entirely on observations of the real world. Mathematics is made by men and has all the fallibility and uncertainty that this implies. It does not exist outside the human mind, and it takes its qualities from the minds of men who created it.
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> Because mathematics is made by men and exists only in their minds, it must be made or re-made in the mind of each person who learns it. In this sense mathematics can only be learnt by being created. We do not believe that a clear distinction can be drawn between the activities of the mathematician inventing new mathematics and the child learning mathematics which is new to him. The child has different resources and different experiences, but both are involved in creative acts. We want to stress that the mathematics a child knows is, in a real sense, his possession, because by a personal act, he has created it.” (pp.1/2)

There are a number of important issues here. The basic epistemological stance respects the child’s creative power, which is similar in principle to the adult, and the mathematics produced is valued. Mathematical situations lead to experimentation, so the process of communicating the mathematics to others and refining the ideas is an essential part of the experience which helps to build confidence, autonomy and relate the new mathematics to that already known. Another significant aspect of the new epistemology is the use of concrete materials as a mediator for mathematical activity. For example, the Cuisenaire Rods can be regarded as a concrete re-presentation of the rational numbers. In this way, it is possible to have visual and tactile experiences which enable us to talk about the relationships between the rods with quite simple language (‘same’, ‘larger than’, ‘half of’, etc.), and to focus on many properties which are fundamental to abstract algebraic relations. Mathematical ideas can emerge when we are engaged with concrete materials, because every perception or action derived from the concrete duplicates itself in mental imagery which becomes structured. Thus the most important objects for mathematical study are the relations between perceptions and actions (Servais, 1970). From the early 1960s onwards a number of ATM publications debated and refined these ideas.

Notes on Mathematics for Children (ATM 1977) developed the earlier position, claiming that mathematics begins with bodily actions, perceptions and speech, and that any pedagogical approach has to acknowledge these facts. Furthermore, our mental powers are fundamentally algebraic by nature and the idea of transformation as a natural capacity of the human mind becomes a unifying algebraic concept. Language and visual perception depend on processes of transformation, and we can recognise that the awareness of transformation can becomes a technique for dynamically exploring the articulation of mathematical situations. It was also recognised that while this idea is very powerful, it is not necessarily the only tool for learning mathematics.Another significant aspect of developing this new methodological approach to teaching is the deliberate focus on a respect for the autonomy of the learner. By now, a clear shift in emphasis had been made; from teaching as an almost independent activity of the teacher, to the acknowledgement of the nature of learning processes so that learning and teaching were now regarded as inextricably linked.

Some links with the present

In this paper I have tried to indicate the most significant aspects of the nature of the changes that occurred in the establishment of new attitudes to teaching and learning in this short period. Not all members of the new community agreed with all the principles that have been discussed here, nor were they necessarily able to put them into practice all the time. However, there was sufficient consensus for a large number of teachers to subscribe to the central issues and develop them through refining their pedagogical practice. Also, I have not given any account of the important work of Caleb Gattegno[7] whose enigmatic statement “only awareness is educable” brought many to focus on the fundamental somatic experiences which lie at the root of our human mental functioning. The intuitive beliefs of teachers, based on their sensitive observation of children, the inspiration of Gattegno, and the philosophy of Lakatos brought together a particular community of practice and encouraged teachers to pursue their ideal, but drew little support from the wider research community, who at the time, were more concerned with quantitative cognitive approaches to investigating classrooms.

In recent papers (Rogers 2001a, 2001b), I have explored some ways in which the symbolism of algebra developed over time from iconic representation in elementary arithmetic and geometry. The ability to develop and use symbol systems is the most outstanding faculty of human cognition (Deacon, 1977), and icons are the most basic means by which things can be ‘re-presented’, and hence

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[7] Gattegno published many works on epistemology, psychology and the learning of mathematics and language.A significant summary of his theory can be found in Gattegno 1978.

're-cognised'. The Cuisenaire rods referred to above are an example of an iconic re-presentation of mathematical structure. The crucial idea that the ATM championed was not merely that these, and other pieces of apparatus are clever teaching devices, but that they are used as a focus for presenting problems, and encouraging active discussion in the social context of the classroom, so that mathematics could be "remade in the mind of each person who learns it".

We constantly use icons in teaching, and know the difficulties students have in making a transition from elementary representations to sophisticated symbolic systems. Unpacking this 'semiotic condensation' (Duval 2001) is a way of thinking about teaching and learning. and it is interesting to observe how much these views on the awareness of our somatic nature have in common with the recent work of biologists and cognitive scientists (Maturana and Varela, 1987; Nunez and Freeman, 1999; Lakoff and Nunez, 2000) who claim that mathematics comes form inside each one of us.

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