Towards numeracy as a cultural historical activity system

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This paper moves toward developing a cultural historical activity theory framework for numeracy. In the first part of the paper three kinds of numeracy are defined and distinguished, as follows: visible-numeracy, useable-numeracy and constructible-numeracy. Next, these distinctions are related to research and practice in numeracy; and tensions, contradictions among these formulations are identified and discussed. This then leads to the introduction of cultural historical activity theory and to a concluding discussion relating to numeracy as a cultural historical activity system.

The Australian Association of Mathematics Teachers has argued in its report *Numeracy=everyone's business* (1997), numeracy "involves using some mathematics to achieve some purpose in a particular context". In other words, numeracy cannot be simply nailed down by quoting mathematical tables and arithmetical relationships. Numeracy is about the context in which it is used. However, neither is it adequate to drop mathematics from the picture. Numeracy is about using mathematical knowledge. How can these elements be brought together in an overall view of numeracy? How can such a view be made to grapple creatively with technological and cultural developments? These are the kinds of questions the framework for numeracy proposed in this paper hopes to better be able to address.

In the first section I look at numeracy through three lenses each offering alternative and competing views of the terrain numeracy encompasses. Using cultural historical activity theory, I then sketch the main features of a new way of thinking about numeracy. According to this view numeracy is seen as the object of a collective activity rather than as an abstracted given. History and culture matter to numeracy not merely in the extrinsic sense of seeing "it" presented differently, but in the intrinsic sense of creating it and constituting it in new socially and culturally apposite ways.

Numeracy as the visible, the useable, the constructible

Working from a hint in Noss (1998), I start my analysis by looking at three kinds of numeracy: visible-numeracy, useable-numeracy, and constructible-numeracy. First, the term 'visible-numeracy' names the kind of knowledge which is intended when using commonly accepted mathematical language and symbols to formulate mathematical relationships and communicate these to others. For instance, counting or measuring are instances of visible-numeracy precisely because these are paradigmatic instances of what constitutes numeracy. Visible-numeracy is the numeracy as illustrated, for instance, in the tradition of the 3 R's. However, Noss observes that not all numeracy is visible in that some "lies beneath the surface of practices and cultures" (1998:3). This gives a rationale for a second kind of numeracy which I call 'useable-numeracy'. Associated with this

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is Noss' notion of numeracy "defined by its use" (p3). 'Useable-numeracy' is the kind of numerical knowledge exhibited when a person is engaged, for instance, in real life (ie non-mathematical) problem solving. Examples of this type of numeracy are typically found in workplace situations. A third kind of numeracy is generated by the concept of constructibility. 'Constructible-numeracy' names that kind of numeracy produced by an individual/social constructive process usually in a learning situation. Indeed, teaching situations in which a teacher/peer facilitates a learner's constructions of mathematical concepts offer typical illustrations of this kind of numeracy.

Note, I do not wish to imply that the distinctions I am making here are clear cut - typically we move among these senses of numeracy as we change roles from learner to teacher to worker to curriculum designer, etc, not necessarily remaining cognisant of the different kinds of numeracy we implicitly invoke. For instance, in the case of teaching the concept of addition, the concept first appears as visible-numeracy to the teacher as a formal part of the curriculum. However, as the teaching methods engage the student in attempting to solve a practical problem, it appears, however momentarily, as useable-numeracy. And finally, in the light of the student's progress towards modifying, developing or extending existing knowledge, the concept of addition appears as constructible-numeracy. This example shows that 'addition' can be viewed as an instance of potentially three different kinds of numerical knowledge. What I intend to argue in this paper is that much of what makes numeracy interesting, challenging and important has to do with the ambiguity of its status among the senses of visibility, useability and constructibility. More precisely, I want to show that these kinds of numeracy come into conflict with each other and that the ensuing tensions give rise to double-bind situations at the heart numeracy as developed here.

In the following subsections I explore the kinds of numeracy identified above in more detail.

Numeracy as the visible: visible-numeracy

The principle of visibility accords with the commonsensical view that numerical knowledge is typically visible as numerical knowledge. A corollary to this view is that if you cannot see evidence of numeracy as expressed in the usual ways, this means that numerical knowledge is not used. What you see is what you get. An important point is that from this point numeracy does not disguise itself, for instance, in the form of knowledge relating to the operations of equipment or technology, the procedures of protocols of a workplace situation, or the received values and practices of a community engaged in working with a common purpose. Instead, numeracy presents itself explicitly in the form of mathematical statements, and mathematics content knowledge topic areas. In the first of these, statements make use of mathematical symbolism and abstraction; in the second, lists of topic areas in mathematics are compiled and grouped in such a way as to form a canonical body of mathematical knowledge.

Other examples of visible-numeracy can be found in typical mathematics textbooks and curriculum documentation. An instance of the later is the Australian Education Council's 1994 report entitled *Mathematics - a curriculum profile for Australian schools*. In this work the authors set out a list of content areas such as number, measurement, space, etc, each with various topic areas such as counting, measuring, etc.

Numeracy as the useable: useable-numeracy

The principal of useability accords with the belief that numeracy stands in an enabling relation to tasks, problems or situations. On this view, numeracy is the knowledge defined by its use in dealing with tasks and problems occurring in every day situations and the workplace. This is the numeracy of everyday life.

In her now classic study, Sylvia Scribner, showed that when we use numerical concepts we not only adapt them to a specific context, but transform them depending on that context (Scribner, 1997). As she followed dairy workers around their daily routines, she observed that opportunistic features of the real life working environment co-construct the kind of numerical knowledge the workers actually implemented in their workplace tasks.

More recently, Jean Lave has investigated the useable numeracy of ‘just plain folks’ in supermarket contexts (1988). Nunes, Schlieman, and Carraher (1993) and Saxe (1988, 1991) have studied numerical proficiencies of candy sellers on the streets of South American cities. Each of these studies have provided powerful examples of useable-numeracy and illustrated that, when compared to visible-numeracy, useable-numeracy is highly adaptable and capable of modification.

In the following examples, I develop these ideas further. My data derives from real-life work situations observed in over-the-counter transactions within the hospitality industry. My purpose is to illustrate the way useable-numeracy engages specific tools and technologies, rules and procedures, common values and protocols for work, evident in particular workplace sites. Transcripts involve interactions among client services offices (CSO), clients (C) and the researcher (R) and illustrates how client accounts are constructed in the workplace.

<u>Episode A</u>

R: And up there you’ve got - oh, your room charges.

CSO: Yes this is all the room charges and room number. Because (in the computer) you've got the charge table (depending on) what rate you (can) check in. And the ultimate charge [inaudible] and we charge according to the number of people in the room. You can go in and change it - for example, (charge for a double) which is for two people and we have the single charge which is for one person in a double room.

R: So do you refer to this much?

CSO: No, its sort of habit to you, once you know it. It's really - yeah I guess its up there for newcomer really. Yeah, and just in case people aren't sure, then they can go through and check it. Once reception staff once they use it, they get (to remember) what charge is what.//

Episode B

R: Do you then enter that into the computer?

CSO: Well first we have to go directly behind me which is to our `Bible'. Now, this bible has to be correct at all times. `Cause we go by the whole motel with this. So hopefully this is always correct. I do have a room available for them [inaudible] so I'll have to put them in, otherwise someone else will let that room go. [inaudible]
OK. I've got them booked in to a suite. So therefore I have to enter into the `suite/double/ or single'. It'll come up a charge type. So there's two occupants. [inaudible] staying in room 102. That's getting in to the normal rates and charge tables. [inaudible] If he was going to get 5 percent discount because he's a corporate cardholder or something we would then go to `1', press [enter] and it changes the Rte straight away.

In these episodes charge variables include: room configuration (single, double, or suite); number of people in a room; whether a discount applies. In constructing an account the CSO must choose values for these variables. Furthermore, some codes for these variables formalise naturalistic meanings eg if three people share a suite, then the does reflecting this number is '3', whereas other codes adopt arbitrarily imposed signifiers eg if a corporate discount applies, the CSO must signify this with the numerical code '1'. This means the CSO must manage a range of variables and codes crossing numerous domains - some visibly numerical, others not. Throughout these operations the CSO must manipulate numerical variables, but know when and where to visit characteristically numerical meaning upon them. In other words, the worker must know which aspects of a presented situation require outright quantification, which require codification though not quantification, and which can be (or need to be) ignored. These examples illustrate that for even apparently straightforward workplace tasks, useable numeracy proficiency requires more than the visible manipulation of arithmetic values.

These examples show that useable-numeracy is complex, and deeply embedded in the context in which it acquires meaning. In the next section I look at the theme constructibility.

Numeracy as the constructible: constructible-numeracy

As indicated previously, constructible-numeracy corresponds to that kind of numeracy defined in virtue of being purposefully constructed. In order to develop the theme of constructibility, it is helpful to consider key developments in theories of learning mathematics education. Two lines of development have been highly influential. In the first, now largely abandoned by scholars (though not necessarily teacher practitioners) numeracy is typically associated with developing the habits of numerical thinking ie the building of bonds or associations as in the work of Thorndike (1922). This approach sees learning numeracy as a behaviouristic phenomenon in which the most important feature was the inculcation of salient responses to given stimuli. Rote learning of numerical relationships, so much a feature of traditional school experience, is consistent with this view. On this view context plays and extrinsic role in the learning process.

In the second line of development, learning is identified with the learners ability to use prior learning and the learning context to formulate meanings for the manipulations which make up learned numeracy experience (Brownell, 1947). This focus on building adequate meanings for learning mathematical concepts and processes has been variously theorised by Gestalt psychologists (Wertheimer, 1945), and cognitive psychologists such as Piaget, Bruner (1960), Dienes (1961, 1967), Steffe, von Glasersfeld (1995) and Cobb (1992, 1994). Because many of these cognitively based theories share the view that the learner actively constructs new knowledge by modifying, extending, replacing and transforming existing knowledge, they have come to be known by the well known umbrella term ‘constructivism’. On the constructivist view, learning numeracy corresponds with the purposeful construction of powerful meanings (ie those that deliver a problem solving outcome) in a given learning context. Whilst much has been left out in this story, the central point is that learning numeracy and constructing numerical knowledge adequate to a problem giving situation are seen as one and the same process. Because the learnable is the constructible and the constructible is the learnable, constructible - numeracy, constructible-numeracy in my parlance, is identified as the outcome of the learning process.

Now the switch to the constructible, the so called "constructivist turn" in mathematics education involves a valuing of context and the conditions of learning mathematics as intrinsic to the learning process. This, in turn, has given rise to views of learning mathematics in which social, historical, cultural and economic contexts are integral rather than peripheral to learning processes. It follows from this analysis that examples of constructible-numeracy are generated by those views of numeracy which emanate from studies which assume and support and understanding of the socially constructed nature of numerical knowledge. Examples of the view that numeracy is the result of social construction include: the anthropological studies of Gerdes, (1985), Bishop, (1988), Saxe (1991) and Nunes, Schlieman & Carraher (1993); the sociological and linguistic studies of Bloor (1983), Lave (1988) and Solomon (1989); the micro-sociological analyses of Bauersfeld (1991), Voigt (1993) and Cobb (1994); and, the discourse theory approaches of Walkerdine (1988, 1994).

In the next section I examine how these different kinds of numeracy stand in relationship to each other.

Conflicts and dilemmas

Over the last 20 years there has been a rich literature demonstrating the tensions between what I have called above visible-numeracy, useable-numeracy and constructible-numeracy. In the previous section I began to illustrate these. Here I am only able to offer a brief survey. Starting for instance with the publication in 1982 in Britain of the Cockroft report (DES, 1982) which had the task of considering "the mathematics required in further and higher education, employment and adult life generally". The Report observed that workplace practices seldom demand standard arithmetic operations such as 2/5 +3/7 and that the need for algebra, let alone such mathematical ideas as proof, modelling and mathematical rigour, was almost nil; it concluded with the (today quite extraordinary) view that "it is possible to summarise a very large part of the mathematical needs of employment as a feeling for measurement". In other words, the thrust of the Cockroft report demonstrated that a focus on useable-mathematics reduces the range of visible-mathematics and, by implication, vice versa. Further, there is a considerable body of evidence that workers make use of mathematics without needing to or wishing to make this fact visible (Wolf, 1984; Strässer, 1999).

Further, Noss (1998) has argued that the problematic relationship between what is identified here as visible-numeracy and useable-numeracy, gives rise to a number of curriculum paradoxes along the following lines. If we merely restrict our view to the "surface of arithmetical activities" in adult working lives, then we are bound to find only "traces and shadows" of mathematics in actual use. The paradox arises when such a finding is used to conclude that the mathematical needs of adult life are feint, for such a conclusion is bound to activate a utilitarian principle relating to the usefulness of school mathematics and see school mathematics become more and more narrowly defined and thus, paradoxically, less and less useful. In terms set up in this paper, concentration on the interests of visible-numeracy oversimplifies issues relating to useable-numeracy, and this leads to numeracy becoming less "useable" that would otherwise be the case.

In a second paradox, Noss suggests educationalists are right to identify a lack of mathematics confidence and alienation within the community. However, in attempting to address this issue, school mathematics has tended to shift towards what is more easily learned and away from "its broader roots in science and technology". Thus, at the cost of making it learnable, it has been disconnected from what is "genuinely useful". If learnability can be equated with constructibility (as I have suggested above), Noss' point is that the very amenability of constructible-numeracy, is a trap. The tendency in schools will be to teach what is most easily constructible - and this opens a possible zone of tension between numeracy which is useable though not so easily constructible, and numeracy which is more easily constructible though less useable.

These paradoxes illustrate that the development of numeracy is governed by highly problematic situations. Focus on any one of visible-numeracy, useable-numeracy or constructible-numeracy tends to exclude the interests of the others - yet it is only this broader set of interests which addresses the needs of students, industry and mathematics. In discussing a way forward, my analysis moves towards developing a cultural-historical view of numeracy. My central claim is that it is in just such a theoretical environment as it offers, that the double-bind situations described by Noss can best be understood.

Numeracy as a cultural historical object

Cultural historical activity theory follows in the tradition of Vygotskian psychology and offers a way of conceptualising the transformation of human systems of production (Leont'ev, 1981; Engeström, 1987, 1991, 1999; Scribner, 1997; Suchman, 1996; Wertsch, 1981, 1985). Social interactions and the role of cultural artefacts in the mediation of activity are at the heart of this theory. An activity system is a social entity which uses cultural artefacts to mediate is common primary motive or driving force. In the activity theory jargon, this motive is known as the 'object' (it is used in the sense of the word 'objective' or purpose). For Engeström (1987, 1991, 1999), cultural artefacts including tools and instruments (physical and psychological), rules and protocols, community, and the division of labour associated with practice, multiply mediate activity. As an activity system progresses, human subjects take up operations and goal directed actions, and outcomes are generated. These relations are depicted in Engeström's model structure of an activity system depicted in Figure 1.

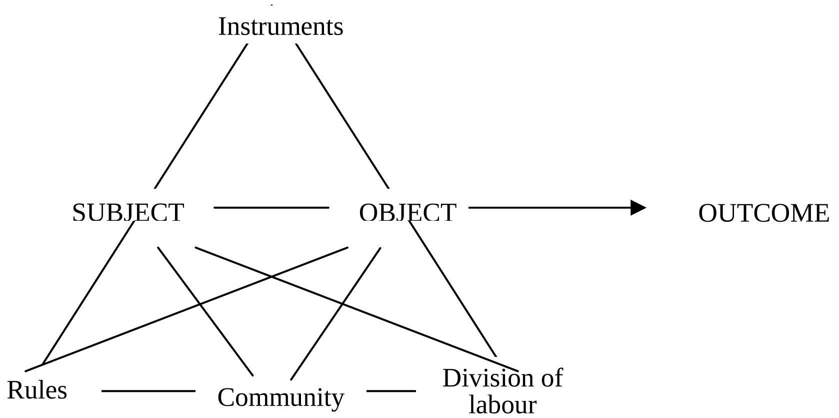

Figure 1: Engeström's model of a cultural historical activity system

Three key features of the theory relate to the development ('expansion' or 'remediation' are jargon terms also used) of the object. These are as follows. First, contradictions within practice are construed to generate the creative impetus for the historico-cultural development of the system. Activity theory is a theory of crisis. The primary impetus for historical development is derived from the need to resolve crises induced by conflicting tendencies, double-bind situations and paradoxes in the nature of the activity system. Second, the object or motive is doublesided in the sense that it both orientates the subject in a direction for action and, in turn, is shaped by the multiple domains of mediation constituting the activity. Simultaneously, actions lead to the continual revision and reconstruction of the object in the light of intermediate goals being reached and the recognition of obstacles to progress. And third, an activity system faced with the need to resolve crises struggles to develop into a "culturally more advanced form" by the process of "remediation". For Engeström, this is process whereby the system makes use of new mediating artefacts (see Figure 2) in order to facilitate desired outcomes.

Analysis of the preceding sections suggests that numeracy is cultural historical activity system whose object is numerical knowledge. Figure 2 outlines features of such a conceptualisation.

The paper concludes with the following comments. At the heart of the activity theoretic framework is a transformation of our understanding of the tensions among visible, useable and constructible numeracies. These should not be viewed as extrinsic eventualities, that is, potentially correctable by suitable means or ways of thinking about numeracy. Instead they are better seen as intrinsic to the nature of numeracy in its current state of cultural development. In other words, Noss' double-bind situations are not anomalies to be overcome so much as keys to understanding the cultural basis of numerical activity. In activity theory language, these anomalies afford primary contradictions underscoring efforts to move numeracy in any given direction.

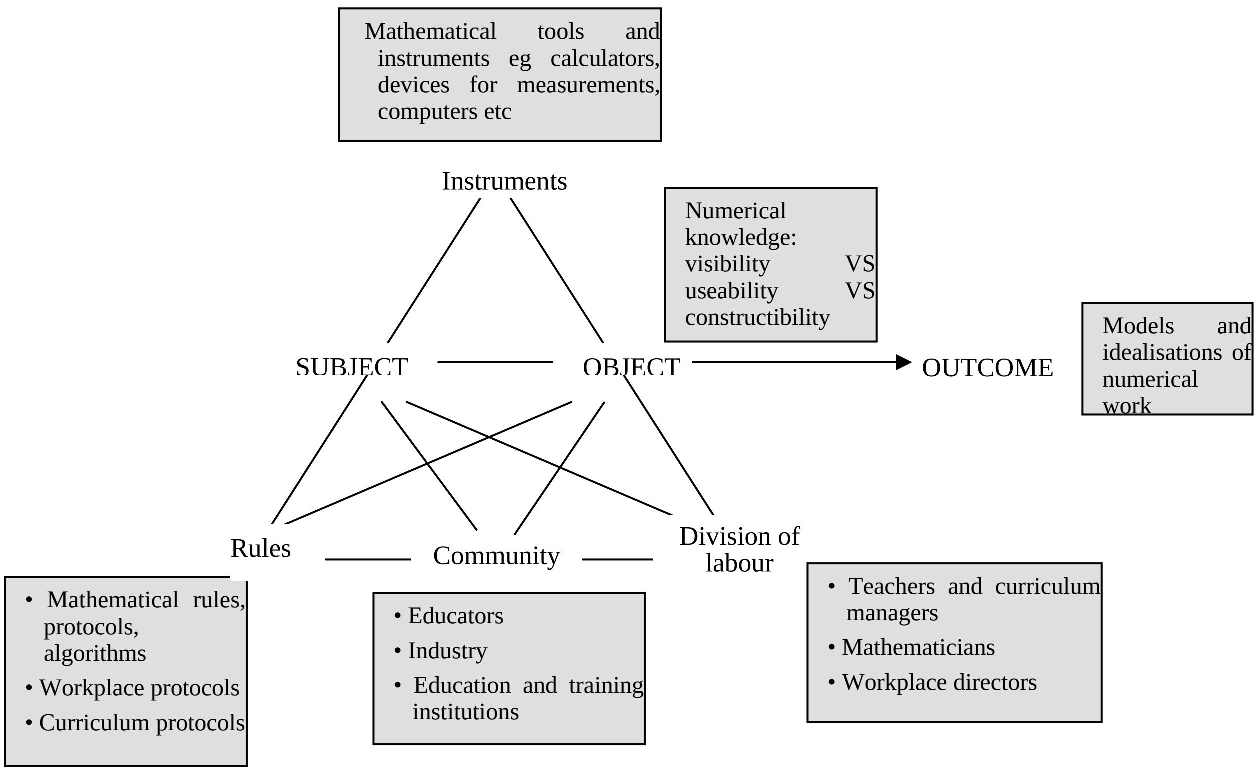

Figure 2: Model of numeracy as a cultural historical activity system

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