Primary school mathematics: An inside view

Tamara Bibby, King's College, UK

This paper is concerned with the nature of primary school maths. It takes as its starting point understandings of maths from a group of generalist primary school teachers. Through a consideration of one of the dominant metaphors (maths as a tool or tool box) it begins the development of an epistemology of maths from the standpoint of the teachers. Notions of efficiency, formal and informal methods and rules, formulae and algorithms that emerged from the data are considered. The ways in which teachers are excluded from 'official' epistemologies of mathematics as well as from more process based conceptions of the subject are also considered.

Introduction

In the UK there is a generally high level of agreement assumed about what we mean when we talk about school maths. Despite an extensive literature on the maths curriculum and mathematical content (eg: Beishuizen & Anghileri, 1998), pedagogical issues (eg: Boaler, 1997b), and the understandings that children bring to maths (eg: Nunes & Bryant, 1996) there is very little discussion about the nature of primary school maths (PSM), the underlying philosophy: what is mathematics?, what are we teaching? While some development of these questions can be found implicitly in the context of other concerns, little is explicit and what there is tends to focus in secondary schools.

Descriptions of epistemologies of maths and the maths curriculum are generally written by mathematicians and educators in positions of power and authority. While teacher discourses of general pedagogy tend to reflect the discourses found in official documentation, the same cannot be said of maths where official and practitioner discourses are markedly different (Bibby, 2001). Through an exploration of the effects of teachers negotiating between conflicitng ideologies this paper addresses the question: what might an epistemology of PSM look like if it were developed from the standpoint of the teachers? Throughout I use the term epistemology narrowly to connote teachers' implicit beliefs about maths. The epistemology of school mathematics developed here is constructed through an exploration of the teachers' experiences, values and beliefs.

The analysis reported here developed from a study in which I conducted a series of between 5 and 7 structured and semi-structured, in-depth interviews with each of seven primary school teachers. The 40 interviews were taped and transcribed. The study used ethnographic tools to explore generalist teachers' personal and professional relationships with maths. The teachers I worked with (Alice, Annie, Elizabeth, Helen, Janet, Joanne and Owen) were all self selected and were not chosen to be typical in any way. Their teaching experience ranged from 2 to 20 years, they taught classes from Nursery to Year 6. Annie, Helen

P. Valero & O. Skovsmose (2002) (Eds.). Proceedings of the 3rd International MES Conference. Copenhagen: Centre for Research in Learning Mathematics, pp. 165-174.

and Joanne were maths co-ordinators, none had a maths degree although Annie had an ‘A’ level in the subject.

In constructing an understanding of maths the teachers I interviewed made much use of their personal and professional experiences of the organisation and management of the curriculum. It appears that personal constructions of the nature of PSM assembled from these experiences were partial, contradictory, and inchoate (Eraut, 1994:111). Making sense of such fragments of bricolated conceptions is problematic. Two dominant metaphors of PSM emerged from the data: maths as a hierarchy which builds on the basics and maths as a tool. In this paper I focus on the latter and use it as a heuristic device. I will focus on notions of utility that emerged in the data; not only maths as a ‘tool box’, but also characteristics of good or appropriate use of these tools: efficiency, speed, the use of rules, formulae and algorithms, and formal and informal methods. Although I would suggest that the analysis undertaken may be applicable beyond the group of teachers interviewed I have no direct evidence for this other than resonances found in the literature which are highlighted.

Mathematics as a ‘tool box’ and how to use it properly

A tool is something that we use to do a job – a means to achieving an end. To be useful, a tool must be appropriate for the job at hand, we must know how to use it, and we must be able to use it.

Tools/‘the basics’	‘Higher’ levels
• To know mathematical facts, principles, and algorithms • To develop a systematic approach to problem solving • To perform computations with speed and accuracy • To develop an awareness of the importance of maths in everyday life	• To become interested in maths • To understand the logical structure of maths • To develop an attitude of inquiry • To develop an awareness of the importance of maths in science and statistics • To understand the nature of proof

Table 1: Cards used with Annie in ‘aims of maths’ interview and her sorting of these

During one interview the cards above were used to prompt teachers to think about the aims of teaching maths. These cards, which used statements on the aims of maths from the SIMS research (Robitaille, 1992:41) were later adapted as some proved difficult to use with words like ‘proof’ causing confusion. The headings on the table above come from Annie; the terms tools and ‘the basics’ were hers and she used them interchangeably. She has ordered the cards from most to least important within the columns. I have chosen to focus on her (and the other teachers’) use of the notion of ‘tools’ as these seemed to relate more closely to beliefs about maths rather than beliefs about pedagogy. Annie has the most clearly articulated set of beliefs about maths being a tool; the others share at least some of her beliefs, if not her conviction.

Annie: [the basics] are the most important things when you're teaching, this is what people need ... they need to know that mathematics is a tool ... you use it as a tool to help you get by in everyday life and if you have a special interest in mathematics then these things ['higher' levels] would come afterwards.

For her the motivation for doing maths comes from the recognition of a problem, so an awareness of the uses of maths in everyday life is seen to be of prime importance, 'everyday maths' is synonymous with 'maths as a tool':

Annie: it sort of underlies all of this ['the basics']. That you know that mathematics is there for you, it's important, it's extremely useful. It's this tool aspect, maths is a tool for you to use and if you don't know that it's there for you to help you then you won't use it.

Despite her initial confident reaction to the cards, later Annie questioned whether the ordering she had used would always be appropriate. She indicated that aims will vary according to the age of the children being taught and their abilities. Further, there was a sense that aims needed to be revised down in the face of children who lack motivation. For the less able those aspects of maths which might be considered to provide intrinsic motivation will need to be pruned away - maths for these children can be pared back to 'the basics', the rest is frills. There would seem to be a paradoxical suggestion that all children should learn the same maths but that only the most able be taught to use the tool - the others just need to learn what the tool is - raising questions about whether such a 'tool for the less able' is really a tool at all?

The metaphors of maths as a tool or a tool box are interesting. The notion of a tool box conjures up an image of a limited closed and contained space. This has clear implications for maths; although with use you may get more familiar and confident with it, it is finite, once you've 'got it' there is no more to learn:

> Personally I am all for numeracy and am a whizz at mental arithmetic. I know my tables forwards and backwards because they were belted into me by an old sadist whose methods worked for a fast learner and devout coward like myself, ... By the end of primary school I was as numerate as I would ever need to be and my five years of maths thereafter were largely a waste of time, at least for me. (MacBeath, 1998)

One 'tool' aspect of maths is as an aid to survival in 'the real world', as such the focus tends to be on money and measures. As Joanna explained: *I presume actually that a lot of people don't go much further than using money in the supermarket and things like that*. However, a tool can also be a pleasure to use:

Annie: For most people [maths] will never be much more than a tool but it's, try and make it a tool that's a good sharp tool, that can be fun to use and it's nice to use - like a sharp knife that cuts well. 'This is great' it makes your life easier if you can do it, if you can operate mathematically to a certain level.

Like the hierarchical nature of maths, its usefulness as a tool is seen as an unproblematic 'common sense' fact. Indeed these aspects of maths were the only ones talked about, the values implicit in many uses of maths (ballistics, demographics, actuarial probability etc.) were not mentioned. Only Joanna seemed aware that the curriculum delivered in school might not map very well onto ‘everyday life’, she pointed out wryly: *we probably wouldn’t have a curriculum if we worried about what people were going to do with it afterwards!* Viewing maths chiefly as a tool for everyday life may have some limitations and consequences for the curriculum and teaching. It certainly precludes any possibility of maths being a tool for maths, for problems to arise within the subject which might be open to investigation for their own sake.

Using the tools well

As indicated above, a tool isn’t a tool unless you can use it in some way. Characteristics of maths as a tool emerged as themes from the analysis of teachers’ constructs of maths; four are explored in this section: efficiency; speed; formal and informal methods; and rules, formulae and algorithms.

The notion of efficiency is so much a part of the culture of business and the market that is currently dominating education in the UK that it seems innocuous and commonsensical. In the discourses that circulate in the public domain and emanate from government, mathematical efficacy is a necessary condition for the efficiency of the workforce and hence the economy. Within maths it is linked to effectiveness which appears to be about getting ‘right answers’. Notions of efficiency have become embedded in the mathematical zeitgeist, *numerate pupils should calculate accurately and efficiently* (DfEE, 1999:4) and *the calculator is a powerful and efficient tool* (p8). One might ask: efficient for whom, or efficient for what purposes? The standard algorithm for long division, for example, while widely taught as an efficient algorithm might not be very efficient applied in complex ‘everyday’ situations. Indeed, as the last quote from the National Numeracy Strategy (NNS) (above) indicates, using a calculator is often most efficient, (which it certainly is for most long division) and yet this is definitely not widely acceptable as a general rule in school. In the context of school maths efficiency seems have a multiplicity of meanings; it may equate with ‘quickest’, or ‘easiest to mark’.

In common with research findings on the attitudes of students (McLeod, 1994), most of the teachers’ comments refer to **the importance of speed**; the fact that one ought to be able to do maths quickly and efficiently and that if it can’t be done quickly then it can’t be done at all (Owen: *I just keep thinking that I've got to do things quicker and then when I can't do it quickly you feel like you can't do it*). This belief would militate against developing perseverance and may be encouraged by an over-emphasis on the testing of rapid recall. With an over-emphasis on speed, accuracy is forfeit:

Elizabeth: I do know how to do it but it’s going to take me a while to put it on paper and I’m going to go wrong because I tend to rush it on paper and then I usually make silly mistakes.… and I’ll get the wrong answer.

The existence of informal, non-school or ‘child’ methods are well documented (eg: Plunkett, 1979; Carraher, et al., 1990) as is adults’ reluctance to use such methods in situations in which they expect school maths to be more useful (Lave, 1988). Formal (school) maths is seen as somehow more proper than informal and practical maths, it seems to be associated with growing up and being fully initiated into school - part of the formality of school beyond the ‘play’ of the early years. Formulae seem emblematic of formal maths. During an interview in which we did maths together and despite being able to correctly calculate percentages mentally with confidence, Joanna was haunted by a formula she had forgotten. She commented that at school she had it written in the back of her attendance register as she was required to calculate attendance percentages regularly. In my house, deprived of her register, she wanted to remember that formula and used trial and improvement methods (unsuccessfully) to reconstruct it. She persisted with this and kept returning to it at odd moments throughout the interview even when we had moved on to other matters. When I challenged why she was doing this she explained:

Joanna: I want to know the exact, if that’s the right way to do it. So I’ve been sitting here thinking. I just want to prove to myself even though I can see [the way I did it mentally] was the right way. … I’d fiddle until I got it. I know it should be there and I feel it’s something I should have … at my fingertips.

The notion that knowing is better than working out is powerful and provides an example of the triumph of hope over experience, as well as illustrating the potency of school maths’ veneration of the formula as an indispensable tool.

At the heart of these beliefs lie others about **the use of rules, formulae and algorithms**. Notions of the importance of these are deeply linked to developing speed and efficiency and to preferences for formal methods. The teachers all believed that procedures and the rules for using them are key to both ‘the basics’ and the conception of maths as a tool box: *maths is basically governed by a set of rules* (Elizabeth); and *[maths] is more a set of rules that have to be followed in order to achieve things* (Owen). How these rules are understood is not clear. They do not relate to axioms in a mathematical sense as truths that act as starting points for the development of other maths, but rather form ends in themselves. Talk of rules, formulae and algorithms seemed interchangeable and to relate to the following of a series of preordained procedures. Notions of ‘rules’ here differs from what might be expected in more formal mathematical contexts (the rules of proof for example).

The belief in the importance and usefulness of a formula was seen by the teachers to transcend the real inefficiency of not being able to remember it, or of needing to have a note of it somewhere. That all children (able or not) have trouble recalling routine procedures is well documented (Brown, 1982:455). A further implication of what was said was that you can trust a formula (an algebraic expression of a generalisation) despite the fact that you can’t trust a rule (a discursive expression of a generalisation) which seems a strange thing to say. However, on reflection the ‘rules’ primary teachers use are acknowledged to be unreliable rules of expediency: an example might be ‘multiplication makes bigger'. In this case perhaps it is unsurprising that these teachers seem to have developed a separation between rule and formula.

For most of my interviewees, the experience of having to learn these rules themselves as children was traumatic. There were feelings that rules had to be learned by rote or they would not be properly learned and that the interviewees would not trust themselves to be able to use them correctly in exams. Conflict arose when parents helping at home used methods different from the 'approved' school method. Elizabeth's concern that *everything* had to be right binds answers tightly to procedures - without *the right* procedure, the answer cannot be right (even if it is correct).

Elizabeth: I never did it [my parents'] way - I always did it the way the school did it because I was worried - I don't know what would have happened if I'd started doing decomposition the other way round - putting your little numbers in different places - I suppose because I was worried I'd have to do everything right it would seem to be wrong even if I got the right answer

Knowing and working out

Many teachers try to compensate for a child's lack of understanding by 'drumming in' rules and algorithms by rote. The fallacy of the belief that this will help them is powerful and persists in the face of evidence of its inefficacy (Brown, 1982:460).

As noted above, knowing tends to be privileged over working out. This flies in the face of the current rhetoric coming from research (Gray, 1991) and policy (DfEE, 1999) which stress the importance of using the two to feed each other. However, shifting perceptions and practices is likely to be problematic and to take more than policy declarations. The problem would appear to lie in the tension between regarding maths as a hierarchy and the utilitarian conception of maths as a tool. The teachers' view of maths as a pyramidal hierarchy tends to privilege knowing. A utility/tools vision would expect the prior stages to be useful and available to the craftsman. While a hierarchy or pyramidal conception, or even a 'step by step' up the ladder conception implies leaving the lower reaches behind and moving on and up, utility requires the carrying of the tool-box. The 'double whammy' from the hierarchical belief in the classroom is the association of 'the basics' with previous years which are also identified as (relatively) babyish and to be left behind in the rush to grow up.

Knowing the tricks

That such tensions are real and lived was expressed in the interviews. Faced with the question: Consider the number $M = 3^3 \times 5^2 \times 7$, Is M divisible by 7? mixed feelings were expressed:

Joanna: [I feel] there was something that I ought to know by looking at whatever M is. I ought to be able to tell without working it out whether it is divisible by 7

That rules, formulae and algorithms are ends in themselves is not enough. How these might be understood or applied also requires consideration and relates to the degree to which the teachers felt they had, or lacked, control of these tools. 'Tricks' was a theme that emerged frequently during data coding. Joanna felt she lacked access to useful tricks:

Joanna: I feel there should be another way - that I'm quite capable of working it out doing it like that [the long way] but I can't see the tricks

Speed and efficiency aren't just about knowing formulae – they are also knowing 'tricks'. There appear to be contradictory feelings about tricks. Is a 'trick' cheating or a valuable short-cut? Interestingly Elizabeth placed the notion of tricks (which she called 'cheats') against strategies. Tricks require no maths, strategies do. There seems to be no mathematical link between what is being done in the 'nine times table finger trick' and the number of fingers we have - there is no point wondering why this trick works, mysteriously it just does. She expressed ambivalence about letting children use the trick/cheat.

Self doubt was endemic, and was most clearly expressed by both Alice and Joanna. Prompted by self-doubt, Alice wanted a 'trick', rule or algorithm when she (correctly) assumed she was missing a piece of knowledge. Having started to explain how she would tackle the question on divisibility (above), and confused by the presence of 'squared' and 'cubed' numbers in the problem, she changed tack apparently prompting me to help out, she believed the trick, the 'obviously easier way', would get her beyond a point of exposure:

Alice: or is there an obviously easier way to do it than that?

Tamara: what do you think? (Alice sighs heavily – long pause) The thing about obviously easier ways is that if you don't know them they're not obvious or easy

Alice: yeah - I think if I knew the facts about - the rules - (goes on to work out the answer the 'long' way)

Notions of tricks and cheats suggest a lack of control over the tools and a use that somehow lies outside maths. To extend the metaphor, it would seem that the tool can best be used to complete its function if it is disguised or wielded in an unorthodox way perhaps tricking the maths into working.

What counts as mathematics

Questions are raised in considering the issues highlighted above: do teachers ever question their images of maths, might they have other conceptions of it? are there cracks in the armour? Although it needs some comment, I do not want to spend long on this section. It is not the content of the curriculum that I am interested in – in England it is currently non-negotiable. Indeed most of the curriculum content would appear to be securely recognised by the teachers as maths (although they reported children to be reluctant to see non-numerical topics other than shape as maths). The NNS is undoubtedly generating some confusion about what counts as maths, or perhaps validating doubts:

Elizabeth: Before I did my subtraction topic I did quite a big thing on 3D shape and nets and looking at tetrahedrons and going beyond just the basic cuboids. I think it's really very important especially because quite a few of my low achievers in number work can feel an element of success in the shape work because I think it's a very different sort of way of looking at maths isn't it? You can be quite spatially aware but not be able to sort of get your head round the whole number thing. … On the other hand, is it really maths? … I know it is officially mathematics but when they keep saying 'numeracy' it does seem to be looking at phasing those sorts of things out [and] I think it would be a real shame to lose out on some of those things, if they were to be devalued.

So there are apparently aspects of the curriculum that offer hope to the non-numerically inclined. But does the fact that some of the less able, less numerically inclined can cope with shape work signify that it is in some way less deserving of a place in the curriculum? Or that it is peripheral? Elizabeth's comment that it is *officially mathematics* would imply it may have a marginal place in the wider scheme of things; perhaps the down-playing of shape, data handling and so on by the NNS confirms her private feelings. The extent to which the structuring of the curriculum, within which the teachers work, compounds their epistemologies remains open to question.

Discussion

That maths might be perceived in ways other than that described here was not entertained by the teachers I interviewed. A debate about other forms of curriculum organisation and different teaching methods which focus on pupil empowerment in the face of maths is conducted within the academy. Such debates extend some way beyond the academy to teachers closely associated with it through their participation on courses and research projects but is not widely engaged with. Further, issues of social justice in the context of maths teaching are discussed largely in the context of secondary schools (eg: Angier & Povey, 1999) while work with younger pupils is largely confined to America with its different traditions and working culture (eg: Lampert, 1990). The extent to which the teachers I interviewed were aware of other epistemological possibilities is unclear but would appear to be limited. Helen who as a maths co-ordinator had been on courses at the local teachers' centre was, at least theoretically, committed to what she called an 'investigative approach'. However in practice she empathised with her colleagues who were increasingly unable to use such an approach in the face of performative aspects of their jobs. Similarly, Joanna had been on courses where the tutor demonstrated another way of being with maths which had a profound effect on her:

Joanna: [The tutor] was good on awe and wonder. ... You know, like mystic roses and all that sort of thing. And showing how you could work out all these huge, big formulas and how it made sense and it was logical and all those things … And he'd bring children in sometimes and do things. It was just like really good how he stretched children's thinking, led them on to the next stage of his questioning instead of just saying - no you do this

Despite Joanna's claim that the *huge, big formulas* made sense, the sense it made is not clear and indeed feels as if it might have been a temporary insight. However, since I did not challenge her to elaborate or explain this comment I cannot be sure. From my standpoint the teachers appear to be outside the maths I might want them and those they teach to be able to engage with. It seems to me that they are being excluded or kept out and this in turn raises questions about how and why they are being kept out: is it a result of actions that they themselves take? Is it a result of the maths that they have been exposed to? Or is it something inherent to the discipline? It would appear that the answer is that all of these things contribute to the situation.

The teachers certainly compound their exclusion from less product based conceptions of maths by their adherence to such epistemological beliefs. That they chose not to engage with process based epistemologies is partly due a lack of opportunity. As I said, these discourses and their warrants tend to circulate within the academy (although Helen and Joanna have certainly had some exposure to such ideas). Looking to their personal histories and experiences learning maths (Bibby, 1999), product focussed maths and an emphasis on tools and utility (as well as the hierarchical nature of the subject with its concomitant focus on the 'basics') was the overwhelming experience for all the teachers I interviewed.

Within the discipline there lie other traps. Many mathematicians use the language of maths loosely when talking publicly perhaps in an attempt to diffuse fear and appear less threatening or perhaps because these 'public' usages are also available to them as members of the public. That official policy documents in the UK are increasingly product focussed will not help, and that the education system itself is similarly focussed on products (examinations) can only act to compound matters. That maths as it is currently portrayed within schools is excluding many learners is well documented (eg: Boaler, 1997a; Boaler, 2000). That the currently dominant discourses of maths are also excluding other, significant groups of the population, including generalist primary school teachers, is less clearly articulated. That many generalist teachers live with an epistemology of maths that necessarily casts them as in some way deficit is problematic to say the least.

Conclusion

This paper explores the metaphor of maths as a tool which emerged from the interview data to begin to develop a teacher-based epistemology of PSM. As I said in the introduction, as an emergent epistemology it is necessarily inchoate. My aim was to consider the ways in which maths was understood by the generalist teachers in my sample. The picture that emerged accorded strongly with reports found elsewhere in the literature. Mathematics was seen largely as numerical topics which might be useful in everyday life; speed, accuracy and abstract language were of the essence. Implications of the tension which exists between simultaneously held conceptions of maths as a tool box are explored elsewhere (Bibby, 2001).

The understandings of PSM that the primary school teachers I interviewed were operating with appear to be more tightly bound into public discourses of maths than to those described by the 'official' texts although the two focus on product/absolutist conceptions of maths. That they are in some way trapped within their mathematical discourses is of concern to those, like myself, who work with teachers. However, the teachers themselves seemed unaware of the tensions which I saw as problematic in their talk. Much as teacher educators might wish to transform the mathematical discourses of generalist teachers, we must recognise that they have to manoeuvre themselves in/between (or against) the competing values inherent within the discursive landscape of 'official' maths and, critically, that they hold their own views based on personal experiences as learners and teachers and professional development. Developing such inchoate epistemologies will be far from straightforward either ethically or practically. However, the task remains critical for teachers, and learners at all levels.

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